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S&P 500 under Dynamic Gordon Model

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S&P 500 under Dynamic Gordon Model*

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Abstract
In this paper, we extend the Dynamic Gordon Model (Campbell and Shiller, 1988) with a semi-structural, medium-term macroeconomic model. The proposed framework allows us to analyze the relationship between output gap, inflation, stock prices, and interest rate as in Blanchard (1981). We estimate the model using Bayesian techniques on quarterly data from 1984 to 2006. The decomposition of the unconditional variance of the variables shows that (i) demand shocks are relevant for both macroeconomic variables and stock prices; (ii) supply and interest rate shocks affect, to a lesser extent, inflation and interest rates, respectively; and (iii) shocks to the price-dividend ratio account for around 30% of the variability of the output gap, inflation, and interest rates, reflecting in this manner the importance of stock prices on the dynamics of macroeconomic variables.

Resumen
En este artículo, extendemos el Modelo Dinámico de Gordon (Campbell y Shiller, 1988) con un modelo macroeconómico, semi-estructural de mediano plazo. El marco de análisis propuesto nos permite analizar la relación entre la brecha de producto, la inflación, el precio de las acciones, y la tasa de interés como en Blanchard (1981). Estimamos el modelo utilizando técnicas Bayesianas con datos trimestrales del período 1984 a 2006. La descomposición de la varianza incondicional de las variables muestra que (i) los shocks de demanda son relevantes tanto para todas las variables macroeconómicas del modelo como para el precio de las acciones; (ii) los shocks de oferta y de tasa de interés afectan, en menor medida, a la inflación y las tasas de interés, respectivamente; y (iii) los shocks a la razón precio-dividendo explica cerca del 30% de las fluctuaciones en la brecha de

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producto, inflación y tasas de interés, lo cual refleja la importancia del precio de las acciones sobre la dinámica de variables macroeconómicas.
1 Introduction

Gordon (1962) model is widely used for evaluating stock prices. The static version of this model provides a closed-form solution for the intrinsic value of a stock, which is determined by three key elements, namely future dividend payments, the dividend growth rate, and the relevant interest rate. However, the assumptions required for achieving the static version of the model are not realistic, neither useful for proper dynamic analysis. In this sense, Campbell and Shiller (1988) provide an alternative version, which is based on a first-order approximation of the log stock return. This alternative is known as Dynamic Gordon growth Model (DGM), and it has been applied to the US stock market to decompose the impact of dividend growth and interest rates over stock returns (Campbell and Ammer, 1993), or to analyze the impact of changes in monetary policy on equity prices (Rigobon and Sack, 2003; Bernanke and Kuttner, 2005), among other empirical applications. Nevertheless, the DGM lacks key economic variables such as output gap or GDP growth. In that sense, Blanchard (1981) provides a theoretical framework in which output gap, stock prices, and interest rates are included. This framework is based on the IS-LM model, which assumes that prices are fixed and, therefore, the output gap is determined by the aggregate demand.

In this paper, we propose to enrich the DGM with a rational expectations New-Keynesian macroeconomic model composed by an aggregate demand equation (or IS curve), an aggregate supply equation (or Phillips curve), and an equation that describes the dynamics of the short-term interest rate. We calibrate the model to assess its main properties and, then, we estimate it for the US economy. Our results show that an unanticipated monetary contraction —that increases the short-term interest rate— reduces the price-dividend (P-D) ratio on impact, but this initial effect dissipates as stock prices escalate. Further, a positive shock to inflation increases the short-term interest rate to reduce higher future inflation, which in turn curtails asset prices through a discounting effect. Lastly, a positive shock to stock prices —that can be associated, for instance, with an increase of risk appetite— implies a positive output gap.
The estimation of the model poses some challenges that we discuss in detail. In particular, the close-to-non-stationary dynamic of the P-D ratio suggests that results based solely on empirical approaches, such as VAR, should be taken with caution. Thus, we use Bayesian techniques to estimate the model using quarterly data from 1984 to 2006, i.e., the period previous to the Global Financial Crisis of 2007-2008. Overall, the estimated model provides a good representation of the dynamics of key variables, although some sample moments related to the inflation rate are over-estimated. The decomposition of the unconditional variance of the variables in the proposed model shows that demand shocks are relevant for both all macroeconomic variables —explaining between 50% to 70% of the total variance— and stock prices. Meanwhile, supply and interest rate shocks affect, partly, inflation and interest rates, respectively. Finally, shocks to the price-dividend ratio account for roughly 30% of the fluctuations of the output gap, inflation and interest rates, hence highlighting the importance of stock prices on the dynamics of macroeconomic variables.

The paper is organized as follows. Section 2 presents the model, whereas Section 3 describes the elements involved in the estimation using a Bayesian approach. Lastly, Section 4 concludes.
2 Model Setup

The model consists of a set of four equations that we describe in detail in this section. The first three equations describe macroeconomic variables such as output gap, inflation, and the short-term interest rate. The last equation, on its part, defines the stock price dynamics. For simplicity, we assume that all variables are expressed as linear approximations around their long-run trends. Thus, they are all demeaned or detrended.

2.1 Equations

The aggregate demand equation, or IS curve, establishes that output gap ($y_t$) is a function of a mixed backward-forward specification, which allows for both inter-temporal optimization and some degree of inertia. The short-term real interest rate ($r_t$) has a negative impact on output gap through the investment channel, whereas the log P-D ratio ($z_t$) captures the positive impact of investment opportunities on $y_t$. Thus,

$$y_t = \lambda_0 y_{t-1} + (1 - \lambda_0)E_t(y_{t+1}) - \lambda_1 r_t + \lambda_2 z_t + u_{1t}, \quad (1)$$

where $u_{1t}$ is a structural aggregate demand shock. The lag and lead parameters of the output gap are forced to sum up to one, which is a standard constraint applied in practice to reduce the number of parameters.

The aggregate supply is a hybrid Phillips curve, where the inflation rate ($\pi_t$) is a rational expectations solution with inertia given by

$$\pi_t = \delta_0 \pi_{t-1} + (1 - \delta_0)E_t(\pi_{t+1}) + \delta_1 y_t + u_{2t}, \quad (2)$$

where $u_{2t}$ is a structural aggregate supply shock. As in the case of equation (1), we consider the same constraint on the lag and lead parameters of $\pi_t$.

To complete the macroeconomic model, we include an equation for the short-term interest
rate \((i_t)\). This equation relates the short-term interest rate with its own lag, the expectations about future inflation, and the output gap, as follows:

\[
i_t = \kappa_0 i_{t-1} + \kappa_1 E_t(\pi_{t+1}) + \kappa_2 y_t + u_{3t},
\]

(3)

where \(u_{3t}\) is a structural short-term interest rate shock. It should be noted that we do not impose any particular restriction on the parameters of this equation because, in our empirical application, we use a short-term corporate-bond interest rate. Also, the short-term real interest rate is defined as follows: \(r_t \equiv i_t - E_t(\pi_{t+1})\).

Finally, we consider a dynamic pricing equation that relates stock prices with expected future dividend payments in a similar way than Campbell and Shiller (1988). We use a log-linear approximation of the log stock return, which is \(x_{t+1} = k + \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t\), where \(p_t\) is the log of stock price and \(d_t\) is the log of dividend payments, both in real terms, and \(k\) and \(\rho\) are parameters of the Taylor approximation. The parameter \(\rho\), on its part, is related to the long run level of the price-dividend ratio.

Because stocks and short-term bonds are assumed to be perfect substitutes, the arbitrage condition implies that they have the same expected real short-term rate of return, i.e., \(E_t(x_{t+1}) = r_t\) (Blanchard, 1981). Thus, ignoring constant terms \((k = 0)\), we have that \(p_t = d_t + \rho E_t(p_{t+1} - d_{t+1}) + E_t(\Delta d_{t+1}) - r_t\). Further, we define \(z_t = p_t - d_t\) as the log P-D ratio, and we also assume that dividends are related to the real economy as \(\Delta d_t = \phi_2 y_t + w_t\), where \(w_t\) is an error term. Taking together all previous elements, a rational-expectations arbitrage equation is given by \(z_t = \rho E_t(z_{t+1}) + \phi_2 E_t(y_{t+1}) - r_t\). However, various related studies have documented a high persistence of the price-dividend ratio in small samples (see Lettau and van Nieuwerburgh, 2008; Chevillon and Mavroeidis, 2018; Golinski \textit{et al.}, 2018; among others). Thus, to incorporate some degree of inertia in the behavior of the P-D ratio, we consider a hybrid version of the arbitrage equation instead:

\[
z_t = \phi_0 z_{t-1} + \phi_1 E_t(z_{t+1}) + \phi_2 E_t(y_{t+1}) - r_t + u_{4t},
\]

(4)
where $u_{4t}$ is a structural shock to the log P-D ratio. To complete the model, we assume that each structural shock follows an uncorrelated, zero-mean AR(1) process as

$$u_{it} = \rho_i u_{i,t-1} + \sigma_i v_{it},$$

for all $i = 1, \ldots, 4$, and $v_{it}$ is a zero-mean, unit-variance shock.

### 2.2 Key IRFs Under Calibration

Before entering into the estimation of the model, we analyze its properties using calibrated parameters. Hence, this section describes in detail the calibration and the results for a set of key Impulse Response Functions (IRFs).

Many parameters of the semi-structural macroeconomic model are well known from the monetary policy literature. Thus, in particular, we consider the calibration used by Fuhrer (2010) in his study of inflation persistence in the US economy. More precisely, for the case of the IS equation, we set $\lambda_0 = \delta_0 = 0.5$, which provides a balance between backward- and forward-looking dynamics of the output gap and inflation. Also, we use $\lambda_1 = \delta_1 = 0.1$, although, in practice, it is possible to have lower magnitudes. In the case of $\lambda_2$, we assume that the P-D ratio has a similar impact on the output gap than the short-term interest rate. For the short-term interest rate equation, we use $\kappa_0 = 0.8$, $\kappa_1 = (1 - 0.8) \times 1.5 = 0.3$, and $\kappa_2 = (1 - 0.8) \times 0.5 = 0.1$. For the price-dividend equation, we consider $\phi_0 = 0.05$ and $\phi_1 = 0.8$. There is no empirical counterpart for $\phi_2$, thus we use 0.5. Finally, we assume for simplicity that $\rho_i = 0.5$ and $\sigma_i = 0.005$ for all structural shocks.

Figure 1 shows the impact of an unanticipated increase in the short-term interest rate that reduces inflation, output gap, and the price-dividend ratio in the short term. The impact on the stock market is driven by both the output gap effect and the discounting effect. In the former case, it is clear that the higher interest rate will deteriorate the output gap for several periods through the investment channel, which in turn will narrow the growth rate of dividends and hence the
P-D ratio. In the second case, the higher short-term interest rate and lower expectations about future inflation imply a higher rate at which future dividend payments are discounted. Therefore, the price-dividend ratio will fall below its steady-state value for several periods.

**Figure 1: Responses to Short-Term Interest Rate Shock**

![Graphs showing responses to short-term interest rate shock](image)

Response of each variable to a one-standard-deviation shock in $i_t$.

Figure 2 shows that a positive shock in inflation will worsen both the output gap and the P-D ratio. This shock can be related, for instance, to an increase of oil prices, thus implying an inverse relationship with stock prices given the predominance of the real activity channel in the short-run. However, in the medium-run, the discounting effect becomes more relevant, and thus, the price to dividend ratio will start to increase because the short-term real interest rate is still below its
steady-state value. In this context, stock prices return to its long-run value about twelve quarters after the shock occurred.

Figure 2: Responses to Inflation Rate Shock

Figure 3 reports the impact of a positive, one-standard-deviation shock in the price-dividend ratio equation. This shock can be viewed, for example, as an increase in the risk appetite of investors. The positive shock in the P-D ratio will stimulate aggregate spending, and lastly the output gap, through consumption and investment (Blanchard, 1981). Specifically, in the former case, since stock shares are part of the wealth, a higher valuation of the stock market will positively affect consumption. In the latter case, a larger price-dividend ratio will boost the value of capital in
place relative to its replacement cost, hence expanding investment.

Figure 3: Responses to Price-Dividend Ratio Shock

Response of each variable to a one-standard-deviation shock in $z_t$. 
3 Estimation

In this section, we introduce the elements for our estimation of the model using a Bayesian approach. First, we describe the data available, then the choice of priors, and we finish the section by discussing various results related to the simulations on the posterior distributions.

3.1 Data

We consider five US aggregate variables in quarterly frequency that span the period from 1984q1 to 2018q4: output gap, P-D ratio, annualized quarterly inflation rate, short-term interest rate, and annualized quarterly dividend growth (Table 1).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Output gap</td>
<td>FRED</td>
<td>HP-filtered real GDP with $\lambda = 1,600$</td>
</tr>
<tr>
<td>$p$</td>
<td>Price</td>
<td>Robert Shiller’s webpage</td>
<td>Log of S&amp;P 500 index divided by CPI</td>
</tr>
<tr>
<td>$d$</td>
<td>Dividend</td>
<td>Robert Shiller’s webpage</td>
<td>Log of dividends per share divided by CPI</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation rate</td>
<td>FRED</td>
<td>Annualized quarterly growth of CPI</td>
</tr>
<tr>
<td>$i$</td>
<td>1-year high quality</td>
<td>FRED</td>
<td>Annual rate</td>
</tr>
<tr>
<td>market corporate bond spot rate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FRED stands for to the Federal Reserve Economic Data of the Federal Reserve Bank of Saint Louis; GDP stands for Gross Domestic Product; CPI stands for Consumer Price Index.

In particular, we compute the output gap ($y_t$) as the difference between real GDP and its HP-filtered trend component, where the smoothing parameter was set equal to 1,600. The price-dividend ratio ($z_t$) corresponds to the difference between the S&P 500 index and dividends per share (in real terms and in logarithms). The dataset is the same as in Shiller (1989) and is available online for download. The inflation rate ($\pi_t$) is the annualized quarterly variation of the Consumer Price Index, whereas dividend growth ($\Delta d_t$) is the annualized quarterly variation of
real dividends per share. The short-term interest rate \((i_t)\) corresponds to the yield on high-quality corporate bonds rated AAA, AA or A, with maturities of up to one year. Lastly, we demean all variables in order to be consistent with the model described in the previous section.

Figure 4: Observables for the Dynamic Gordon Model
In the whole sample, there is a declining trend in the short-term interest rate being below its average during the last decade (Figure 4). On the other hand, the price-dividend ratio has an increasing trend before 2000, followed by some mean-reversion and an important drop during the Global Financial Crisis of 2007-2008 that is also present in other stock price measures that consider cyclical adjustments\(^1\). Similarly, the dividend growth rate has a dramatic fall during the crisis period, where this variable reached levels around 30% below its historical average. Thus, because of the previous discussion, we consider all variables during the pre-crisis period 1984q1-2006q4 in our estimations to exclude the dynamics exhibited by the data during the crisis period that would be otherwise difficult to be explained by the model.

### 3.2 Posterials

As we discussed in the calibration section, several parameters of the model are standard in the monetary policy literature. Thus, we draw priors for this set of parameters from the Beta, Normal, and Inverse Gamma distributions, where the prior mean was set equal to the calibrated values, and the prior standard deviations were chosen to contemplate plausible ranges for the parameters.

Table 2 reports the mode, the mean, and the 90% probability interval of the posterior distribution of the structural parameters obtained by the Metropolis-Hastings algorithm with 10,000 replications. Several aspects are worth highlighting from this table.

First, the posterior mean of the parameters associated with the IS curve is somewhat different from usual calibrations in standard semi-structural DSGE models and the one used in the previous section. The lag parameter of this curve is estimated to be 0.68, which is roughly 1.4 times larger than the calibrated value. Moreover, the output gap is almost insensitive to variations of the short-term real interest rate, whereas the effects of the P-D ratio on the output gap, although being smaller than its prior value, are around four times larger than those of \( r_t \). Second, the estimated parameters of the hybrid Phillips curve suggest a balanced forward- and backward-looking behavior of inflation. Further, our results show also that output gap has relevant effects on inflation. Third, the posterior mean of the parameters associated with the equation for the

\(^1\)For a comparison of these measures, see the Appendix.
short term interest rate shows an important backward-looking component that is consistent with the downward trend exhibited by the 1-year corporate bond interest rate during the whole sample (Figure 4). Also, the point estimate of the parameter $\kappa_1$ is roughly ten times smaller than its calibrated value, implying that the corporate rate is rather insensitive to fluctuations of the expected future inflation. Lastly, the estimates related to the pricing equation support the idea that there exists an interaction between the output gap and the stock market. In particular, the posterior mean of $\phi_2$ is close to 0.8, suggesting that the P-D ratio reacts to changes in the expected value of future output growth.

Table 2: Posterior Estimate of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr.</td>
<td>Mean</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>Beta</td>
<td>0.800</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Beta</td>
<td>0.300</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Beta</td>
<td>0.050</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Beta</td>
<td>0.800</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>Normal</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>Beta</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Inv. Gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Inv. Gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>Inv. Gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>Inv. Gamma</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The posterior distribution was obtained using the Metropolis-Hastings algorithm with 10,000 replications. PI stands for probability interval.
3.3 Moment Comparison

To evaluate the performance of our proposed model, we compare some key sample moment conditions of the output gap, inflation, the P-D ratio, and the short-term interest rate, with their corresponding moments derived from the estimated model. In particular, we focus our attention on the volatility, the first-order autocorrelation, and the cross-correlation between these variables.

Table 3: Moment Comparison

<table>
<thead>
<tr>
<th>Moment, Correlation</th>
<th>Point Estimate</th>
<th>Std. Err.</th>
<th>Model Moment</th>
<th>z-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_y )</td>
<td>0.010</td>
<td>0.009</td>
<td>0.031</td>
<td>2.27</td>
</tr>
<tr>
<td>( \sigma_\pi )</td>
<td>0.004</td>
<td>0.003</td>
<td>0.036</td>
<td>10.7</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>0.401</td>
<td>0.049</td>
<td>0.311</td>
<td>1.87</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>0.005</td>
<td>0.004</td>
<td>0.016</td>
<td>2.35</td>
</tr>
<tr>
<td>( \varphi_y(1) )</td>
<td>0.892</td>
<td>0.049</td>
<td>0.901</td>
<td>0.16</td>
</tr>
<tr>
<td>( \varphi_\pi(1) )</td>
<td>0.373</td>
<td>0.103</td>
<td>0.962</td>
<td>5.70</td>
</tr>
<tr>
<td>( \varphi_z(1) )</td>
<td>0.980</td>
<td>0.014</td>
<td>0.963</td>
<td>1.19</td>
</tr>
<tr>
<td>( \varphi_i(1) )</td>
<td>0.960</td>
<td>0.022</td>
<td>0.974</td>
<td>0.65</td>
</tr>
<tr>
<td>( \varphi_{y,\pi} )</td>
<td>0.262</td>
<td>0.102</td>
<td>0.523</td>
<td>2.57</td>
</tr>
<tr>
<td>( \varphi_{y,z} )</td>
<td>-0.040</td>
<td>0.105</td>
<td>-0.010</td>
<td>0.28</td>
</tr>
<tr>
<td>( \varphi_{y,i} )</td>
<td>0.579</td>
<td>0.086</td>
<td>0.180</td>
<td>4.64</td>
</tr>
</tbody>
</table>

\( \sigma \) denotes volatility, \( \varphi(1) \) denotes first-order serial correlation, and \( \varphi_{a,b} \) denotes the cross-correlation between variables \( a \) and \( b \). Model moments based on the posterior mean of the estimated parameters.

Table 3 shows the results of our estimations. Overall, we note that the model is capable of capturing some of the univariate and multivariate dynamic behavior of the endogenous variables mentioned before. For instance, in our dataset, the price-dividend ratio is about 40 times more volatile than the output gap. In comparison, the estimated model delivers a P-D ratio that is roughly 10 times more volatile, although this difference is not statistically relevant at standard significance levels. Analogously, the sample autocorrelation of the output gap and the P-D ratio, and the cross-correlation between them is close to 0.89, 0.98 and -0.04, respectively, whereas the estimated model predicts values for these statistics of around 0.90, 0.96 and -0.01. However,
the model fails to replicate some moments related to the inflation rate. More concretely, the volatility and first-order autocorrelation of $\pi$, and the correlation between $y$ and $\pi$ predicted by the estimated model are significantly above their sample counterparts.

### 3.4 Variance Decomposition

Figure 5 shows the decomposition of the unconditional variance of the output gap, inflation, the short-term interest rate, and the price-dividend ratio based on the mean of the model posterior distribution shown in Table 2.

![Figure 5: Unconditional Variance Decomposition](image)

From this figure, we note that fluctuations of the output gap are primarily driven by demand and stock price shocks, thus reflecting the importance of the rather backward-looking nature of this variable, and the importance of stock prices on aggregate demand through consumption and investment. A similar situation occurs when decomposing movements in inflation, which are propelled mainly by demand and stock price shocks and, to a lesser extent, by supply shocks.
In the case of interest rate shocks, they are less relevant on interest rates in general, where their incidence is about 3% of the total variance of this variable due to their low volatility and persistence. Finally, in the case of the P-D ratio, roughly all variations are explained by demand shocks, while stock price shocks account for about one-tenth of the total variance. To understand this result, we should look at the prior and posterior mean of $\phi_2$. From our estimations, the sensitivity of the price-dividend ratio to changes in the expected value of future output gap is significantly higher than its prior value, which is in line with the fact that, previous to the Global Financial Crisis of 2007-2008, episodes of positive output gaps tend to coincide with stock market expansions.

4 Conclusions

In this paper, we extend the Dynamic Gordon growth Model (DGM) with a medium-term, semi-structural macroeconomic model. Thus, the proposed four-equations framework allows us to analyze the relationship between output gap, inflation, stock price, and interest rate as in Blanchard (1981). Calibrated with literature-based parameters, our DGM implies that an unanticipated monetary contraction depresses stock prices, meanwhile, a positive shock in the price-dividend equation—that could be associated with an increase in the risk appetite of investors, for instance—implies a positive output gap. Further, the estimation of the model using Bayesian techniques supports most of the calibrated parameters. However, it fails to fit some sample moments of the inflation rate. Lastly, the decomposition of the unconditional variance of the variables in the model shows that demand shocks are relevant for both all macroeconomic variables and stock prices. Moreover, shocks to the price-dividend ratio account for around 30% of the variability of the output gap, inflation, and interest rates, hence highlighting, in turn, the importance of stock prices on the dynamics of macroeconomic variables.
References


A Appendix

A.1 Price-Dividend versus Cyclical-Adjusted Measures

In our model, we consider the elements of the P-D ratio in logs and demeaned. However, it is common in the financial industry to use an alternative measure by considering the price-earnings ratio, where earnings per share are smoothed over a moving window of ten years. This alternative measure of price pressures is called the Cyclical-Adjusted Price to Earnings (CAPE) ratio.

Figure 6: Comparison of Alternative Price Ratios

Figure 6 shows the P-D along with CAPE and the Cyclical-Adjusted Price to Dividend (CAPD) ratios, the latter being the traditional price-dividend ratio but with dividends per share smoothed over a moving window of ten years, all in logs. Note that, prior to the crisis period, all three indicators behave similarly. However, after that event, the smoothed ratios tend to exhibit a positive trend, in sharp contrast with the P-D ratio that remains relatively stable. This result suggests that CAPE and CAPD are strongly affected by the crisis period, where both earnings and dividends per share decreased significantly (Figure 4).
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