Asymmetric monetary policy responses and the effects of a rise in the inflation target

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Asymmetric monetary policy responses and the effects of a rise in the inflation target

Benjamín García
Banco Central de Chile

Abstract

The effective lower bound (ELB) on interest rates introduces an explicit non-linearity for feasible monetary policy paths: interest rates cannot go below a certain rate. In a forward looking environment, the ELB can affect the monetary policy decisions not only when the bound is reached, but also when there is a possibility that the bound may be reached in the future. In this context, as a recommendation for monetary policy in a low-inflation environment, Reifschneider and Williams (2002 FOMC) propose an asymmetric Taylor Rule with a threshold level that automatically drives the interest rate to zero whenever they fall below one percent. I test the hypothesis that the Federal Reserve has behaved in a manner consistent with Reifschneider and Williams’ advice, finding evidence of a negative correlation between the level of the interest rate and the strength of the monetary policy response. Using an estimated nonlinear DSGE model, I show that a monetary policy which act symmetrically and asymmetrically can have significantly different consequences. In particular, I study the relevance of this behavior for the analysis of a permanent rise of the inflation target.

Resumen

El límite inferior efectivo para las tasas de interés (ELB por sus siglas en inglés) introduce una no-linealidad explícita para las trayectorias posibles de política monetaria: las tasas no pueden bajar de cierto nivel. En un escenario con agentes que miran a futuro, el ELB puede afectar las decisiones de política monetaria no solo cuando el límite es alcanzado, sino que también cuando existe una posibilidad de que el límite se alcance en el futuro. En este contexto, como recomendación de política monetaria en un ambiente de bajas tasas de interés, Reifschneider y Williams (2002 FOMC) proponen un regla de Taylor asimétrica, con un gatillo que hace caer las tasas a cero en el momento en que bajan del uno por ciento. En este trabajo testeo la hipótesis de que la Reserva Federal de los EE.UU. se ha comportado de manera consistente con las sugerencias de Reifschneider y Williams. Encuentro una correlación negativa entre el nivel de las tasas de interés y la fuerza con que responde la política monetaria. Utilizando un modelo estocástico de equilibrio general no lineal, muestro que reglas de política simétricas y asimétricas tienen consecuencias significativamente distintas. En particular, estudio la relevancia de este tipo de comportamientos para el análisis de un cambio permanente de la meta de inflación.

* I thank Carl Walsh, Kenneth Kletzer, and Grace Gu for their excellent comments and suggestions. The views in this document are those of the author and do not represent official positions of the Central Bank of Chile or its Board Members.
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1 Introduction

In a forward looking environment, the mere possibility of constraints becoming binding in the future can have effects in current decisions. For monetary policy, one of such constraints is the effective lower bound (ELB) for interest rates. If rates go low enough, the monetary authority could become unable to further stimulate the economy. In this context, Reifschneider and Williams (2002 FOMC) argue in favor of an asymmetric Taylor rule. Their proposal is justified in terms of avoiding excessive welfare costs in cases where the bound on nominal interest rates puts a limit on the possible responses of the monetary authorities against negative shocks. When policy is constrained by a lower bound, prices tend to fall more rapidly, causing an unintended policy tightening as real interest rates rise. As they argue, this ends up exacerbating the rise in unemployment and, under extreme conditions, can even turn into a self-reinforcing spiral, with falling output pushing down inflation, and falling inflation pushing real interest rates higher, putting even more pressure on unemployment.

One possible solution to ameliorate the welfare costs during zero interest rate episodes is to have rules that are more responsive to fluctuations of inflation and output. This type of rule has two main benefits. First, it reduces the probability that inflation falls below the target when a major disturbance hits the economy, removing one of the ingredients of a deflationary spiral. The second benefit is that as it reduces interest rates quickly when the economy weakens, the severity of major downturns is reduced, lowering the risk of deflation.

However, a more aggressive behavior can also have some drawbacks. It can increase the volatility of interest rates and the frequency of policy reversals. It also amplifies the risk and magnitude of policy mistakes in the presence of faulty data and mismeasurement of the economy’s productive capacity. Finally, Reifschneider and Williams argue that a quick drop in interest rates could trigger a confidence crisis if investors become worried that the interest rate could become constrained by
a zero bound.

In this context, they propose an asymmetric rule that triggers a stronger response when the interest rate is close to the zero bound. As the argument goes, the asymmetric rule allows the potential drawbacks of strong monetary responses to appear only when the costs of maintaining a relatively weak policy response grow in magnitude. When interest rates are relatively high, the benefits of a stronger policy, in terms of avoiding welfare costs during zero interest rate episodes, are smaller compared to these costs. Therefore, an increased response to output and inflation is not justified.

2 Empirical evidence of asymmetry on Taylor rule responses

The hypothesis of an asymmetric monetary policy is tested for the US economy through three empirical approaches: a vector autoregression with time-varying policy responses, a single-equation Taylor rule regression with interaction terms, and a DSGE model with a nonlinear monetary policy rule.

2.1 Single-equation Taylor rule estimation with interaction terms

The Taylor rule is estimated as a single-equation regression, where every variable but interest rate is considered exogenous. In order to test the asymmetry of monetary policy responses, interaction terms are added to the standard rule. In the estimated equation, the interest rate \( r \) is a function of the last-period interest rate, inflation \( \pi \), unemployment \( u \), and the interaction between the interest rate and inflation and unemployment.

\[
r_t = \beta_0 + \beta_i r_{t-1} + \beta_\pi \pi_t + \beta_u u_t + \beta_{\pi,i} \pi_t r_t + \beta_{u,i} u_t r_t + \varepsilon_t \tag{1}
\]
Given the specification, if significant parameters accompanying the interaction between interest rate and exogenous variables are found, the magnitude of the monetary policy response to inflation and unemployment depends on the interest rate.

A situation in which the interaction term and direct effect parameters have the same sign implies that the estimated monetary policy response becomes weaker as interest rates approach zero. Opposite signs, on the other hand, signify that smaller interest rates are correlated with stronger policy responses.

Contemporary shocks to the interest rate equation are expected to influence current and expected future values of inflation and unemployment. Potential bias in the estimated parameters due to endogeneity is tackled with two different approaches, an OLS estimation using past periods’ market expectations as proxies for the potentially endogenous variables and a GMM estimation that uses these variables’ lagged values as instruments, in line with Favero (2001).

For the first approach, data from the Survey of Professional Forecasters of the Federal Reserve Bank of Philadelphia are used. The quarterly survey started in 1968, with additional variables added in 1981. It asks participants for their forecasts on the evolution of selected economic variables. The mean of the respondents’ answers is used for the regressions. The proxy for the expected interaction term is computed as the mean of the multiplication of the correspondent answers of each surveyed agent; then, $\hat{E}(xy) = \sum_{j=1}^{J} x_j y_j / J$. where $x_j$ and $y_j$ are the answers of each agent $j$ based on their expectation for variables $x$ and $y$.

The Taylor rule is then estimated using the answers given at $t-1$ for the expected value of the variables for time $t$ or $t+1$, depending on whether the specification tested is forward looking. As expectations were formed in the last period, they are not affected by contemporary shocks to the interest rate equation, and are therefore expected to be uncorrelated with the error term, eliminating the parameter bias by endogeneity.
The expected unemployment rate is used as a measure of economic activity. The expected CPI inflation rate is employed for prices, and the expected 3-month treasury bill rate for the interest rate. While answers regarding unemployment expectations are available from 1968, questions on CPI and interest rates were added to the survey only in the 1981 revision. The sample period for regression is therefore restricted to an initial date of 1981Q4. It is set to end at 2008Q3, the last quarter before the zero lower bound (ZLB) occurs.

For GMM estimation, the endogeneity problem is tackled by using lagged variable values as instruments. Not relying on survey data also allows for an extended sample size. The GMM sample spans the period 1966Q1 to 2008Q3.

The regression results are provided in Table 1. The interaction terms between interest rate and unemployment are consistently significant and show the opposite sign of the coefficient accompanying the unemployment rate. This would indicate a stronger monetary policy response to unemployment shocks when interest rates are close to zero.

Regarding inflation, the OLS specification interaction terms are always significant and have the opposite sign of the direct-effect terms. For GMM estimation, however, both the direct-effect and interaction terms for inflation and interest rates appear significant only in the forward-looking specification, and include dummies for the different Fed chairs. In the specification where the inflation and interaction terms appear to be significant, the coefficients are also comparable in magnitude to their OLS counterparts.

Overall, the results for the single-equation Taylor rule estimation support the hypothesis of a stronger monetary policy response to inflation and output when interest rates are closer to zero, although the results are more robust for unemployment than inflation.
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Table 1: Estimated interest rate effect on the Taylor rule response to inflation and unemployment
2.2 VAR with time-varying coefficients

The hypotheses of an asymmetric monetary policy is also tested through a time-varying parameter methodology. A three-equation time-varying parameter vector autoregression (TVP-VAR) is estimated. Following the Gali and Gambetti (2009) methodology, and similar to Primiceri (2005), Cogley and Sargent (2001, 2005), and Cogley and Sbordone (2008), an $n$-variable and $p$-lag Bayesian VAR is estimated with a specification given by

$$x_t = A_{0,t} + A_{1,t} x_{t-1} + \ldots + A_{p,t} x_{t-p} + u_t$$

(2)

where $x_t$ is a vector of endogenous variables, $A_{0,t}$ is a vector of time-varying coefficients, and $A_{i,t}, i = 1, \ldots, p$ are matrices of time-varying coefficients. The residuals $u_t$ are normally distributed with mean zero and variance-covariance matrix $\Sigma_t$. Let $A_t = [A_{0,t}, A_{1,t}, \ldots, A_{p,t}]$ and $\theta_t = vec(A_t')$, a vector that stack all elements of $A_t$. The parameters from $\theta_t$ are assumed to evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. The residuals $u_t$ are normally distributed with mean zero and a variance-covariance matrix $\Sigma_t$, which is also allowed to change over time$^1$.

The endogenous variables incorporated into the VAR are the federal funds rate, the inflation rate, and the output gap. From the estimation results, a time-varying interest rate response to inflation and output can be obtained:

$$r_t = A_t + \rho_t (L) r_t + \phi_{\pi,t} (L) \pi_t + \phi_{y,t} (L) y_t + v_t$$

(3)

As in Primiceri (2005), the time-varying long-term responses to inflation and output can be expressed as

$$\Phi_{\pi,t} = (1 - \rho_t (1))^{-1} \cdot \phi_{\pi,t} (1)$$

(4)

$^1$A full description of the estimation procedure is presented in the appendix, section A
In order to test for monetary policy asymmetric responses, the time series of $\Phi_{y,t}$ and $\Phi_{it}$ are regressed against the interest rate. In order to interpret the results as percentage changes in the strength of monetary policy responses, the dependent variables are expressed as logarithms.

Table 2 shows the regression results. A significant coefficient accompanying the interest rate implies that the interest rate has an effect on the response strength. If the coefficient is negative, then a lower interest rate correlates with stronger responses. When no dummies for Fed chairs are present, the model is not able to identify any effect. However, after introduction of the dummies, the results consistently show negative coefficients for the asymmetry parameter for both inflation and output responses, suggesting that the monetary policy reaction to both variables tends to be stronger when the interest rate approaches zero. Introducing trend variables does not affect the results significantly.

Interestingly, the magnitude of the effect is very similar for the estimated responses of inflation and output, although the estimates for the output response appear to be more precise, with the estimated standard deviation for the interest rate parameter being 30% to 40% larger in the inflation regression than the output regression.

2.3 DSGE model with a nonlinear Taylor rule

A third approach used to test the presence of asymmetries in the monetary policy responses is based on a Bayesian dynamic stochastic general equilibrium model estimation. The specification is based on a widely used medium-scale DSGE model developed by Smets and Wouters(2007) but with a modified Taylor rule that incorporates the possibility of asymmetric responses.

The original model, belonging to the New Keynesian or New Neoclassical Synthesis class of monetary
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<th>DUM_Burns</th>
<th>DUM_Miller</th>
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N obs: 171  171  171  171  171  171
Adj. $R^2$: 0.991  0.993  0.993  0.996  0.997  0.997

Table 2: Estimated effect of interest rate on the TVP-VAR implied time-varying Taylor rule coefficients

* indicates significance at 10%  ** indicates significance at 5%  *** indicates significance at 1%
business cycle models\textsuperscript{2}, has 14 endogenous variables: output $y_t$, consumption $c_t$, real value of 
capital stock $q_t$, capital services used in production $k_t^s$, installed capital $k_t$, capital utilization rate 
$z_t$, rental rate of capital $r_t^k$, inflation $\pi_t$, wages $w_t$, mark-ups for the goods and labor markets $\mu_t^p$ and 
$\mu_t^w$, worked hours $l_t$, and nominal interest rate $r_t$. It features many frictions that affect both 
nominal and real decisions of households and firms, including sticky nominal price and wage settings, 
habit formation in consumption and investment adjustment costs, and variable capital utilization and 
fixed costs in production. It also includes seven orthogonal structural shocks: total factor 
productivity shocks, two shocks that affect the intertemporal margin (risk premium shocks and 
investment-specific technology shocks), two shocks that affect the intratemporal margin (wage and 
price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

Households maximize a nonseparable utility function with two arguments (goods and labor effort) 
over an infinite life horizon. Consumption appears in the utility function relative to a time-varying 
external habit variable. Labor is differentiated by a union, so there is some monopoly power over 
wages. Households rent capital services to firms and decide how much capital to accumulate given 
the increasing costs of capital adjustment. Firms produce differentiated goods, decide on labor 
and capital inputs, and set prices. The model also features an exogenous spending process, and a 
monetary policy reaction function: 
\[ r_t = \rho r_{t-1} + (1 - \rho) \left[ r_{\pi, t} \pi_t + r_{\hat{y}, t} \hat{y}_t \right] + r_{\Delta \hat{y}, t} \Delta \hat{y}_t + \epsilon^r_t. \] 
The monetary authority adjusts the interest rate $r_t$ in response to inflation $\pi_t$, the output gap $\hat{y}_t$, and the change 
in the output gap $\Delta \hat{y}_t$.

In this paper’s version of the model, the Taylor rule is augmented to allow for asymmetric responses, 
where the interest rate determines the strength of the monetary policy response to the variables:

\[ r_t = \rho r_{t-1} + (1 - \rho) \left[ r_{\pi, t} \pi_t + r_{\hat{y}, t} \hat{y}_t \right] + r_{\Delta \hat{y}, t} \Delta \hat{y}_t + \epsilon^r_t \]  \hspace{1cm} (6)

\textsuperscript{2}A full description of the equations of the model is presented in the appendix, section B.1
where

\[ r_{y,t} = r_y \exp \left\{ m_y r_t / 100 \right\} \]  

(7)

\[ r_{\Delta y,t} = r_{\Delta y} \exp \left\{ m_y r_t / 100 \right\} \]  

(8)

\[ r_{\pi,t} = 1 + (\pi - 1) \exp \left\{ m_{\pi} r_t / 100 \right\} \]  

(9)

The parameters \( m_{\pi} \) and \( m_y \) define the asymmetry level of the responses to inflation and output, respectively. Under this specification, the effect of \( r_t \) on the response strength is constant in percentage terms. A coefficient of 1 on the asymmetry parameter implies that an interest rate 1% above its steady-state level is correlated with a 1% stronger response to the corresponding variable.\(^3\) Conversely, a negative value of the parameter means that monetary policy responses are stronger when interest rates are lower. If \( m_{\pi} = m_y = 0 \), then \( r_{\pi,t} = r_{\pi}, r_{y,t} = r_y, \) and \( r_{\Delta y,t} = r_{\Delta y}, \) and the Taylor rule collapses to the original state from the Smets and Wouters (2007) model.

The structure for \( r_{\pi,t} \) is slightly modified with respect to equations (7) and (8) in order to guarantee the fulfillment of the Taylor principle. As the effect of \( r_t \) on \( r_{\pi,t} \) is defined only on the part of \( \tau \) above unity, \( r_{\pi,t} \) is always greater than 1, regardless of the interest rate.

Given the nonlinear nature of the Taylor rule, a second-order approximation of the model is estimated. Seven observable variables are used for the estimation: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Measurement error is allowed for all observables but the interest rate. A particle filter procedure is used for the nonlinear estimation, with 50,000 particles and 25,000 MCMC replications. Uninformative flat priors are set for the asymmetry parameters. The rest of the priors are kept as in the base model specification. The model is estimated for two samples. The first one starts — as in Smets and Wouters(2007) — in 1966Q1. The second sample

\(^3\)In the model, the interest rate is specified in quarterly terms. If the asymmetry parameter is 1, a 1% interest rate increase in annual terms implies a 0.25% stronger response.
Table 3: Estimated monetary policy parameters of a DSGE model with an augmented Taylor Rule

<table>
<thead>
<tr>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
<th>Prob(x&lt;0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μσ</td>
<td></td>
</tr>
<tr>
<td>ρ</td>
<td>Beta 0.75 0.10</td>
<td>0.87 0.85</td>
</tr>
<tr>
<td>τx</td>
<td>Normal 1.50 0.25</td>
<td>2.03 1.92</td>
</tr>
<tr>
<td>τ̄y</td>
<td>Normal 0.13 0.05</td>
<td>0.17 0.19</td>
</tr>
<tr>
<td>τΔy</td>
<td>Normal 0.13 0.05</td>
<td>0.26 0.17</td>
</tr>
<tr>
<td>mz</td>
<td>Uniform 0.00 173</td>
<td>-132 -50.2</td>
</tr>
<tr>
<td>mz</td>
<td>Uniform 0.00 173</td>
<td>-37.2 -18.2</td>
</tr>
</tbody>
</table>

Table 4: Sensitivity of Taylor rule parameters to the annual interest rate

<table>
<thead>
<tr>
<th>1966Q1-2008Q3</th>
<th>1987Q1-2008Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=1%</td>
<td>r=5%</td>
</tr>
<tr>
<td>rz</td>
<td>3.60</td>
</tr>
<tr>
<td>ry</td>
<td>0.22</td>
</tr>
<tr>
<td>rΔy</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 3 presents the estimation results for the Taylor rule parameters. Evidence of an asymmetric monetary policy response to both output and inflation can be observed. The negative values of the asymmetry parameters $m_\pi$ and $m_y$ are indicative of a stronger monetary policy response when the interest rate is lower. This is particularly marked in the extended sample estimation, with higher estimated coefficients, and virtually all the mass of the parameter distribution on the negative side.

The implications of the estimated parameters on the strength of the monetary policy responses are summarized in Table 4. Focusing on the full-sample estimation, where the steady-state nominal

---

4The prior and posterior distributions for all the model parameters are presented in the appendix, tables 6 and 7.
interest rate is estimated to be 3.8% in annual terms, a movement in the interest rate from 1% to 10% correlates with a change in the Taylor rule response with respect to inflation from 3.60 to 1.13. The same variation in interest rates correlates with a change in the response to the output gap from 0.22 to 0.10 and with the output gap growth from 0.33 to 0.14.

3 Economic implications of an asymmetric monetary policy

In order to assess the economic impact of an asymmetric monetary policy, the DSGE model is simulated under the estimated parameters from both samples. The simulation considered both the baseline cases with the estimated asymmetry parameters, as well as a counterfactual with the asymmetry parameter values set at zero.

The model is simulated using an extended path algorithm, as proposed by Fair and Taylor (1983) and further developed by Adjemian and Juillard (2011, 2013). It allows for occasionally binding constraints, such as the non-negativity of the nominal interest rate. One advantage of the extended path algorithm compared to other popular algorithms that consider this type of constraints, like the one by Guerrieri and Iacovello (2015), is that it can also handle nonlinear models, avoiding linearization altogether. This is a necessary feature due to the nonlinear nature of the model’s asymmetric Taylor rule.

The sample starting from 1987Q4 contains only the great moderation era, with very low volatility shocks. This causes the simulated interest rate paths to seldom reach the zero boundary.

In order to make both specifications more comparable, the simulations for the shorter sample are made with shocks having twice their estimated standard deviation. Figure 1 shows the distribution

---

5 A comprehensive description of the extended path simulation algorithm is presented in Adjemian and Juillard (2013)
of the simulated interest rates under these different parameterizations.

The statistics to be analyzed are the frequency of deep recessions and zero-interest-rate events, as
well as the sensitivity of the ZLB frequency to changes in the long-term inflation level. In order to
compute these numbers, each model specification is simulated for 50,000 quarters. Simulations are
subdivided into 125 subsamples of 100 years each. For every subsample, each statistic is computed.
Reported in table 5 are the sample means and the standard deviations of those means, calculated
by bootstrapping methods. For proper comparability between specifications, and to correctly in-
corporate the covariance into the standard errors of the difference between models, the shocks for
every simulation are generated using the same seed for the random generation process. The same
is done for the bootstrapping random sampling.

3.1 Severity of downturns

The main justification behind Reifschneider and Williams’ proposal for an asymmetric policy rule is
to avoid highly recessive episodes. When interest rates are low, monetary policy preemptively starts
Table 5: Economic implications of an asymmetric monetary policy rule

<table>
<thead>
<tr>
<th></th>
<th>Short sample parameters</th>
<th>Full Sample parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m=m̅ m=0 Δ</td>
<td>m=m̅ m=0 Δ</td>
</tr>
<tr>
<td><strong>Deep Recessions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarters per 100 years</td>
<td>10.9 13.4 -2.5</td>
<td>32.4 56.4 -24.1</td>
</tr>
<tr>
<td></td>
<td>(0.79) (0.87) (0.31)</td>
<td>(2.10) (3.13) (1.88)</td>
</tr>
<tr>
<td>Episodes per 100 years</td>
<td>2.39 2.82 -0.43</td>
<td>3.92 5.29 -1.37</td>
</tr>
<tr>
<td></td>
<td>(0.14) (0.14) (0.07)</td>
<td>(0.23) (0.25) (0.16)</td>
</tr>
<tr>
<td>Mean Episode Duration</td>
<td>4.32 4.55 -0.23</td>
<td>8.04 10.33 -2.29</td>
</tr>
<tr>
<td></td>
<td>(0.19) (0.14) (0.14)</td>
<td>(0.41) (0.47) (0.44)</td>
</tr>
<tr>
<td>E(y</td>
<td>r=0)</td>
<td>-4.04 -4.73 0.69</td>
</tr>
<tr>
<td></td>
<td>(0.17) (0.19) (0.03)</td>
<td>(0.77) (1.02) (0.85)</td>
</tr>
<tr>
<td><strong>Episodes per 100 years</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarters per 100 years</td>
<td>15.7 13.8 1.88</td>
<td>20.1 24.3 -4.17</td>
</tr>
<tr>
<td></td>
<td>(0.90) (0.83) (0.15)</td>
<td>(1.63) (1.84) (0.52)</td>
</tr>
<tr>
<td>Episodes per 100 years</td>
<td>4.70 4.17 0.53</td>
<td>6.45 7.34 -0.90</td>
</tr>
<tr>
<td></td>
<td>(0.23) (0.22) (0.09)</td>
<td>(0.45) (0.49) (0.23)</td>
</tr>
<tr>
<td>Mean Episode Duration</td>
<td>3.27 3.23 0.04</td>
<td>3.09 3.27 -0.17</td>
</tr>
<tr>
<td></td>
<td>(0.11) (0.12) (0.06)</td>
<td>(0.13) (0.13) (0.10)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>+1% inflation → Δ ZLB likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ ZLB Quarters per 100 years</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Δ ZLB Episodes per 100 years</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Δ ZLB Mean Episode Duration</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

*Bootstraped standard deviation in parentheses*

reacting in a stronger manner. By the time the interest rate hits zero and is unable to stimulate the economy, recessive and/or deflationary events will be of a lower magnitude. The rule avoids an excessive downturn that can potentially be amplified if the interest rate hits the minimum and loses its ability to aid economic recovery.

Model simulations support this idea. Episodes of deep recessions are defined as periods with a negative output gap larger than 5 percent. A monetary policy with an asymmetric response to variables decreases the number of episodes by 15 to 26 percent, depending on the sample period of the estimation. The mean duration of each of those episodes decreases by 22 percent if the full-sample period is considered. The Taylor rule asymmetry, according to the restricted-sample
estimation results, reduces the duration by 5 percent.

Overall, the frequency of these episodes decreases by 19 to 43 percent. Also of interest is the fact that, with the asymmetric rule, the mean output gap during ZLB episodes is between 0.7 and 4.0 percent (of GDP) closer to zero. In the simulated model, when the interest rate cannot fall lower, the economy tends to be in better shape if an asymmetric rule is in place.

3.2 Likelihood of reaching the ZLB

An asymmetric rule that responds more strongly to shocks when interest rates are lower has a flip side: the likelihood of their actually hitting the ZLB could in fact increase. Stronger responses can lead to higher interest rate volatility, and therefore a greater probability that interest rates reach their minimum. However, in forward-looking models, a mere threat to respond strongly potentially stabilizes the variable, leading to an interest rate that actually moves less than would be the case with a weaker response.

Both of these scenarios appear, depending on the sample used for the estimation. With the full-sample parameters, an asymmetric Taylor rule response decreases the frequency of a zero-level interest rate by 17%. On the other hand, with the restricted-sample parameters, the zero-lower-bound frequency increases by 14%. In both cases, the change is driven almost completely by a variation in the number of episodes. The variation in the mean duration of the episodes is negligible.

3.3 Inflation level and ZLB frequency

Higher inflation levels are expected to reduce the likelihood of reaching the ZLB. As nominal interest rates will also be higher on average, the restriction on the nominal interest rate will be further away
from the steady state, and therefore will reach its limit less frequently.

The sensitivity of the ZLB frequency to changes in the long-term inflation level is also affected by the degree of asymmetry in the Taylor rule. A higher inflation target is associated with nominal interest rates that on average are further from zero. With an asymmetric policy rule, this also changes the expected strength of the monetary responses. In forward-looking models, the effect on the volatility of inflation and output – and thus interest rates – is not clear, and will depend on the particular structure of the economy.

In the case of a full-sample estimation when an asymmetric Taylor rule is in place, an increase in the inflation target is, at the same time, more effective in reducing the duration of the ZLB episodes and less effective in reducing the number of episodes. In terms of the total reduction in the frequency of an ZLB episode , both effects cancel out each other. With the restricted-sample parameters, an increase in the inflation target is more efficient in reducing the frequency of ZLB episodes but just as effective in reducing the mean duration of each episode. Overall, when an asymmetric rule is in place, the reduction in the frequency of a zero interest rate is 9.5% higher when the Taylor rule presents asymmetric responses to variables.

4 Conclusions

With the recent financial crisis, the possibility of interest rates hitting their minimum feasible levels turned into a real concern, beyond a mere theoretical curiosity. To deal with that possibility, Reifschneider and Williams(2002) propose a non-linear feedback rule for the monetary policy, with a threshold level that automatically drives the interest rate to zero whenever they fall below one percent.
I test the hypothesis that the Federal Reserve behavior has been consistent with an asymmetry similar to Reifschneider and Williams’ proposal. The three empirical methods used are unable to reject the null hypothesis. The implications of this findings are analyzed using a DSGE model with an asymmetric Taylor rule. As predicted by Reifschneider and Williams, the negative consequences of the effective lower bound constraint are reduced. An asymmetric Taylor rule provide less, shorter and less pronounced episodes of recessions. The likelihood of being constrained by the lower bound, however, depends on the model parameters.

Additionally I analyze implications of an asymmetric Taylor rule on the effects of an increase in the inflation target, a policy suggested by Blanchard et al. (2010) as a mechanism to reduce the likelihood that the interest rate would hit the zero lower bound. The results are inconclusive. Depending on the model parameters, an asymmetric Taylor rule may correlate with either an increase or a reduction of the policy effectiveness in terms of reducing the likelihood of reaching the lower bound.
Appendix

A Time varying VAR estimation procedure

Following the Gali and Gambetti (2009) methodology, and similar to Primiceri (2005), Cogley and Sargent (2001, 2005), and Cogley and Sbordone (2008), an \( n \)-variable and \( p \)-lag Bayesian VAR is estimated, with a specification given by

\[
\begin{align*}
x_t &= A_{0,t} + A_{1,t}x_{t-1} + \ldots + A_{p,t}x_{t-p} + u_t \\
\end{align*}
\]  

where \( x_t \) is a vector of endogenous variables, \( A_{0,t} \) is a vector of time-varying coefficients, and \( A_{i,t}, i = 1, \ldots, p \) are matrices of time-varying coefficients. The residuals \( u_t \) are normally distributed with mean zero and variance-covariance matrix \( \Sigma_t \). Let \( \theta_t = vec(A_t') \), a vector that stacks all elements of \( A_t \). The parameters from \( \theta_t \) are assumed to evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. Then, apart from the reflecting barrier, \( \theta_t \) evolves as

\[
\theta_t = \theta_{t-1} + \omega_t
\]

where \( \omega_t \sim N(0, \Omega) \). The variance-covariance \( \Sigma_t \) is also assumed to change over time. Let \( \Sigma_t = F_tD_tF_t' \), where \( F_t^{-1} \) is the lower triangular matrix

\[
F_t^{-1} = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
\gamma_{2,1,t} & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
\gamma_{n,1,t} & \ldots & \gamma_{n,n,t} & 1
\end{bmatrix}
\]
and $D_t$ is the diagonal matrix

$$
D_t = \begin{bmatrix}
\sigma_{1,t} & 0 & \ldots & 0 \\
0 & \sigma_{2,t} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \ldots & 0 & \sigma_{n,t}
\end{bmatrix}
$$

The evolution of $\Sigma_t$ is determined by the evolution of $\gamma_t$ and $\sigma_t$, where the first is a vector of the non-zero and non-one elements of $F_t^{-1}$, and $\sigma_t$ is the vector of diagonal elements of $D_t$.

$$
\gamma_t = \gamma_{t-1} + \zeta_t 
$$

$$
\ln (\sigma_t) = \ln (\sigma_{t-1}) + \xi_t
$$

Where $\zeta_t \sim N(0, \Psi)$ and $\xi_t \sim N(0, \Xi)$

Let $\theta^T, \gamma^T, \sigma^T$ be a sequence of corresponding variables up to time $T$. The conditional prior density is assumed to be given by

$$p (\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) \propto I (\theta^T) f (\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega)$$

where

$$I (\theta^T) = \Pi_{t=0}^T I (\theta_t)$$

$$f (\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) = f (\theta_0) f (\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega)$$

And $f (\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega)$ is consistent with (11). The index function $I (\theta_t)$ equals one if the absolute value of every root from the associated VAR polynomial is larger than one, and zero
otherwise. It ensures that the estimated system will not behave in an explosive way by setting the likelihood that those parameters equal zero. Let $\hat{z}_{OLS}$ be the estimated parameter $z$ from a time-invariant VAR using a training sample with $T_0$ observations. As in Benati and Mumtaz (2007) and Primiceri (2005), the prior densities and parameters take the form of

\[ p(\theta_0) \propto I(\theta_0) N\left(\hat{\theta}_{OLS}, \sigma^2_{\theta_{OLS}}\right) \]  
\[ p(\log \sigma_0) = N(\log \hat{\sigma}_{OLS}, 10 \times I) \]  
\[ p(\gamma_0) = N(\hat{\gamma}_{OLS}, |\hat{\gamma}_{OLS}|) \]  
\[ p(\Omega) = IW\left(10^{-5} \times \sigma^2_{\theta_{OLS}}, T_0\right) \]  
\[ p(\Psi) = IW\left(10^{-1} \times |\hat{\gamma}_{OLS}|, 2\right) \]  
\[ p(\Xi_{i,i}) = IG\left(0.0001, \frac{1}{2}\right) \]  

(17a)  
(17b)  
(17c)  
(17d)  
(17e)  
(17f)

The realizations from the posterior density are drawn using a Markov chain Monte Carlo (MCMC) algorithm that works iteratively, the Gibbs sampler. Each iteration is done in four steps. In each step, realizations of a subset of the parameters are drawn conditional on a particular realization of the remaining coefficients. In the next step, another subset of parameters is drawn conditional on the draws from the previous step. Under regularity conditions, the iterations on these four steps produce draws from the joint density.

For each iteration $i$ of the Gibbs sampler, in the first step, realizations are drawn for $\theta_i^T$ conditional on $x^T, \gamma_{i-1}^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$, and $\Omega_{i-1}$ by the Carter and Kohn (1994) algorithm. In the second step, using the same procedure described in Primiceri (2005), draws of $\gamma^T$ are obtained conditional on $x^T, \theta_i^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$ and $\Omega_{i-1}$. In the third step, draws are obtained from $\sigma_i^T$ conditional
on \( x^T, \theta_i^T, \gamma_i^T, \Psi_{i-1}, \Xi_{i-1} \), and \( \Omega_{i-1} \) with the Jaquier et al. (2004) algorithm. Finally, in the fourth step, draws are obtained from \( \Psi_i, \Xi_i, \) and \( \Omega_i \), conditional on \( x^T, \theta_i^T, \gamma_i^T, \) and \( \sigma_i^T \) as in Gelman et al. (1995). The parameters \( \gamma_0^T, \sigma_0^T, \Psi_0, \Xi_0, \) and \( \Omega_0 \) are initialized using the correspondent parameters of the training-sample estimated VAR.
B Smets and Wouters DSGE model

B.1 Model description

This section describes the equations from the Smets and Wouters (2007) DSGE model that is going to be estimated. The model has 14 endogenous variables: output $y_t$, consumption $c_t$, real value of capital stock $q_t$, capital services used in production $k^*_t$, installed capital $k_t$, capital utilization rate $z_t$, rental rate of capital $r^k_t$, inflation $\pi_t$, wages $w_t$, mark-ups for the goods and labor markets $\mu^p_t$ and $\mu^w_t$, worked hours $l_t$, and the nominal interest rate $r_t$. All variables are log-linearized around their steady-state balanced growth path. Starred variables denote steady-state values.

The aggregate resource constraint is given by:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon^q_y$$

(18a)

where

$$c_y = 1 - i_y - g_y$$

(18b)

$$i_y = (\gamma - 1 + \delta) k_y$$

(18c)

$$z_y = R^k t k_y$$

(18d)

$$\varepsilon^q_t = \rho_g \varepsilon^q_{t-1} + \eta^q_t + \rho_g \eta^q_t$$

(18e)

Output is absorbed by consumption, capital utilization costs that are a function of the capital utilization rate, and exogenous spending. $c_y, i_y, g_y$, and $z_y$ are the steady-state shares of output absorbed by the corresponding variables.
The Euler equation for consumption is given by

\[ c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) + c_3 (r_t + \varepsilon_t^b) \]  \hspace{1cm} (19a)

where

\[ r_r_t = r_t - E_t \pi_{t+1} \]  \hspace{1cm} (19b)
\[ c_1 = \frac{\lambda/\gamma}{1 + \lambda/\gamma} \]  \hspace{1cm} (19c)
\[ c_2 = \frac{(\sigma_c - 1) \left( W_s^b L_s/C_s \right)}{\sigma_c (1 + \lambda/\gamma)} \]  \hspace{1cm} (19d)
\[ c_3 = \frac{1 - \lambda/\gamma}{\sigma_c (1 + \lambda/\gamma)} \]  \hspace{1cm} (19e)
\[ \varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b \]  \hspace{1cm} (19f)

The ex-ante expected real interest rate is \( r_r_t \). \( (1 - \sigma_c) \) and \( \lambda \) come from the underlying utility function and represent, respectively, the intertemporal elasticity of substitution for consumption and a consumption habit parameter. The parameter \( \gamma \) represents the steady-state growth, and the disturbance term \( \varepsilon_t^b \) is a wedge between the interest rate controlled by the central bank and the return on assets held by households.

The Euler equation that defines the dynamics of investment is given by

\[ i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \]  \hspace{1cm} (20a)
where

\[ i_1 = \frac{1}{1 + \beta \gamma^{1-\sigma} c} \quad (20b) \]

\[ i_2 = \frac{i_1}{\gamma^2 \varphi} \quad (20c) \]

\[ \varepsilon^i_t = \rho_t \varepsilon^i_{t-1} + \eta^i_t \quad (20d) \]

\( \varphi \) is the steady-state elasticity of the capital adjustment cost function, and \( \beta \) is the discount factor applied by households. \( \varepsilon^i_t \) represents a disturbance to the investment-specific technology process.

The arbitrage equation for the real value of the capital stock is

\[ q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r^k_{t+1} - rr_t \quad (21a) \]

where

\[ q_1 = \beta \gamma^{-\sigma} c (1 - \delta) = \frac{1 - \delta}{R^k_t + (1 - \delta)} \quad (21b) \]

The depreciation rate is denoted by the parameter \( \delta \).

The aggregate production function comes from a Cobb Douglas function with capital and labor services as inputs:

\[ y_t = \phi_p (\alpha k^a_t + (1 - \alpha) l_t + \varepsilon^a_t) \quad (22a) \]

where

\[ \varepsilon^a_t = \rho_a \varepsilon^a_{t-1} + \eta^a_t \quad (22b) \]

The parameter \( \alpha \) is the share of capital in production, while the parameter \( \phi_p \) is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.
Capital services are a function of the capital stock installed last period and the degree of capital utilization, as it is assumed that capital becomes effective with a one-quarter lag.

\[ k_t^s = k_{t-1} + z_t \]  

(23)

The degree of capital utilization is a positive function of the rental rate of capital:

\[ z_t = z_1 r_t^k \]  

(24a)

where

\[ z_1 = \frac{1 - \psi}{\psi} \]  

(24b)

The parameter \( \psi \), normalized to between zero and one, relates to the capital utilization adjustment cost.

Capital accumulation depends on the net investment, as well as the efficiency of investment expenditure as captured by the investment-specific technology disturbance \( \varepsilon_t^i \):

\[ k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i \]  

(25a)

where

\[ k_1 = \frac{1 - \delta}{\gamma} \]  

(25b)

\[ k_2 = \frac{1 - k_1}{i_2} \]  

(25c)

The price mark-up on the goods market is defined as the difference between the average price and
the nominal marginal cost. It is equal to the difference between the marginal product of labor $mpl_t$ and the real wage:

$$\mu^p_t = mpl_t - w_t \quad (26a)$$

where

$$mpl_t = \alpha (k^s_t - l_t) + \varepsilon^a_t \quad (26b)$$

Profit maximization by price-setting firms gives rise to a New-Keynesian Phillips curve:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu^p_t + \varepsilon^p_t \quad (27a)$$

where

$$\pi_1 = \frac{\iota_p}{1 + \beta \gamma^{1-\sigma} \iota_p} \quad (27b)$$

$$\pi_2 = \frac{\beta \gamma^{1-\sigma}}{1 + \beta \gamma^{1-\sigma} \iota_p} \quad (27c)$$

$$\pi_3 = \frac{(1 - \xi_p) (1 - \beta \gamma^{1-\sigma} \xi_p)}{(1 + \beta \gamma^{1-\sigma} \iota_p) \xi_p ((\phi_p - 1) \varepsilon_p + 1)} \quad (27d)$$

$$\varepsilon^p_t = \rho_p \varepsilon^p_{t-1} + \eta^p_t - \mu_p \eta^p_{t-1} \quad (27e)$$

The price-setting frictions are defined by the parameters $\xi_p$ and $\iota_p$, which respectively define the fraction of firms that are unable to re-optimize prices and the degree of indexation to past inflation. $\varepsilon_p$ defines the curvature of the Kimball goods market aggregator. $\varepsilon^p_t$ is a price mark-up disturbance.

The rental rate of capital is related to the real wage and the capital-labor ratio:

$$r^k_t = w_t - (k_t - l_t) \quad (28)$$

27
The labor market’s wage mark-up is defined as the difference between the real wage and the marginal rate of substitution between labor and consumption $\text{mrs}_t$:

$$\mu_t^w = w_t - \text{mrs}_t$$  \hspace{1cm} (29a)

where

$$\text{mrs}_t = \sigma l_t + (1 - \lambda / \gamma)^{-1} (c_t - (\lambda / \gamma) c_{t-1})$$  \hspace{1cm} (29b)

$\sigma_t$ is the elasticity of labor supply with respect to the real wage.

Due to wage stickiness and partial indexation of wages, real wages only gradually adjust to their desired levels:

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w$$  \hspace{1cm} (30a)

where

$$w_1 = \frac{1}{1 + \beta \gamma^{1-\sigma_c}}$$  \hspace{1cm} (30b)

$$w_2 = \frac{1 + \beta \gamma^{1-\sigma_c} \tau_w}{1 + \beta \gamma^{1-\sigma_c}}$$  \hspace{1cm} (30c)

$$w_3 = \frac{\tau_w}{(1 + \beta \gamma^{1-\sigma_c} \tau_p)}$$  \hspace{1cm} (30d)

$$w_4 = \frac{(1 - \xi_w) (1 - \beta \gamma^{1-\sigma_c} \xi_w)}{(1 + \beta \gamma^{1-\sigma_c}) \xi_w ((\phi_w - 1) \varepsilon_w + 1)}$$  \hspace{1cm} (30e)

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$$  \hspace{1cm} (30f)

The wage stickiness level and wage indexation are represented, respectively, by $\xi_w$ and $\tau_w$. The Kimball market aggregator is $\varepsilon_w$, the steady-state labor market mark-up is $(\phi_w - 1)$, and $\varepsilon_t^w$ is a wage mark-up disturbance.
In the Smets and Wouters model, the Taylor rule gradually adjust the interest rate in response to inflation and output. There is also a short-run feedback from the change in the output gap.

\[ r_t = \rho r_{t-1} + (1 - \rho) [\pi_{t} \pi_t + \pi_y \hat{y}_t] + \pi_{\Delta y} \Delta \hat{y}_t + \varepsilon^r_t \]  

(31a)

where

\[ \hat{y}_t = y_t - y_t^p \]  

(31b)

\[ \varepsilon^r_t = \rho \varepsilon^r_{t-1} + \eta^r_t \]  

(31c)

The output gap \( \hat{y}_t \) is the difference between actual and potential output. Potential output \( y_t^p \) is defined as the level of output that would prevail under flexible prices and wages.

As described in section (2.3) the Taylor rule is modified to allow for an asymmetric response to variables. Equation (31a) then becomes:

\[ r_t = \rho r_{t-1} + (1 - \rho) [r_{\pi, t} \pi_t + r_{y, t} \hat{y}_t] + r_{\Delta y, t} \Delta \hat{y}_t + \varepsilon^r_t \]  

(32a)

where

\[ r_{\pi, t} = 1 + (\pi - 1) \exp\{m_{\pi} r_t / 100\} \]  

(32b)

\[ r_{y, t} = \pi_y \exp\{m_y r_t / 100\} \]  

(32c)

\[ r_{\Delta y, t} = \pi_{\Delta y} \exp\{m_{\Delta y} r_t / 100\} \]  

(32d)

With this modification, the strength of the response to output and inflation is not fixed at \( r_y, r_y, \) and \( \pi_{\Delta y} \), but changes over time as a function of the interest rate \( r_t \).
B.2 Estimation

A second-order approximation of the model is estimated. Seven observable variables are used for the estimation: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Measurement error $\varepsilon_{t}^{\text{obs}} \sim N(0, \sigma_{t}^{\text{obs}})$ is allowed for all observables but the interest rate. The corresponding measurement equation is

$$
Y_t = \begin{bmatrix}
dlGDP_t \\
dlCONS_t \\
dlINV_t \\
dlWAG_t \\
lHOURS_t \\
dlP_t \\
FEDFUNDS_t
\end{bmatrix} = \begin{bmatrix}
\gamma \\
\gamma \\
\gamma \\
\gamma \\
\gamma \\
\gamma \\
\gamma
\end{bmatrix} \begin{bmatrix}
y_t - y_{t-1} \\
c_t - c_{t-1} \\
i_t - i_{t-1} \\
w_t - w_{t-1} \\
l_t \\
\pi_t \\
r_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{t}^{yobs} \\
\varepsilon_{t}^{cobs} \\
\varepsilon_{t}^{iobs} \\
\varepsilon_{t}^{wobs} \\
\varepsilon_{t}^{lobs} \\
\varepsilon_{t}^{\piobs} \\
0
\end{bmatrix}
$$

A particle filter procedure is used for the nonlinear estimation, with 50,000 particles and 25,000 MCMC replications, from which the first 1,000 are discarded as a burn-in period. Uninformative flat priors are set for the asymmetry parameters. For the rest of the parameters, the priors are kept as in the base model specification. The model is estimated for two samples. The first one starts, as in Smets and Wouters (2007), in 1966Q1. The second sample is set to begin at 1987Q4, as Alan Greenspan took charge of the Fed. For both specifications, the end of the sample is set at 2008Q3, just before the federal funds rate reached the zero lower bound. Tables 6 and 7 present the results of the estimation.
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<th>Parameter</th>
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Table 6: Prior and posterior distribution of the DSGE structural parameters
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<td>$\sigma_{lobs}$</td>
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</table>

Table 7: Prior and posterior distribution of the DSGE shock processes and measurement errors
References


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