Substitution of the result from equation (5) into equation (3) yields the amount of non-contingent debt of each maturity:

\[ B_{0t} = \frac{V_0 + \sum_{j=1}^{\infty} P_{0j} \hat{G}_j}{\sum_{j=1}^{\infty} P_{0j}} - \hat{G}_t \]  

(7)

Hence, the amount of debt with maturity \( t \) is the difference between permanent outlay (including the financing of any initial debt) and the certainty-equivalent outlay for period \( t \).

Suppose, as an example, that each period has the same level of certainty-equivalent outlay, \( \hat{G}_t \). In this case, the terms involving the outlays cancel in equation (7), and the quantity of debt for each period is a constant, given by\(^4\)

\[ B_{0t} = V_0 / \sum_{j=1}^{\infty} P_{0j} \]  

(8)

One way to look at this answer, in terms of pure discount bonds, is that the maturity structure of the non-contingent debt has no holes.\(^5\) The government arranges the debt at the outset so that the real amounts to be paid in each future

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\(^4\)If the one-period, non-contingent real interest rate is the constant \( r \), then \( B_{0t} = rV_0 \).

\(^5\)This result on the desirable maturity structure of the public debt therefore differs from the suggestion of Friedman (1959, p. 63): "I can find no valid argument for the present policy of issuing a wide variety of securities ... The alternative suggestion follows ... Issue ... debt in two standard forms, one short-term ... the other moderately long-term. The short security might be a 90-day bill ... The longer security might best be a consol—that is, a perpetuity ... A less extreme break would be to make it, let us say, an eight- or ten-year maturity. I do not myself believe that the precise maturity of the debt outstanding is of great significance.
period (up to $t = \infty$) are the same. However, because of the discounting on future real payouts (that is, a declining time path of the $P_{0j}$), the current market value of the outstanding debt declines steadily with maturity.

From the standpoint of coupon bonds, the government should structure its debt as indexed perpetuities (consols). These issues pay a uniform and perpetual stream of real coupons but have no principal payments.

The prescription for consols may seem to entail a maturity structure of the public debt that is much longer than that observed in practice. However, when governments issue real bonds, the stated maturity—and, more pertinently, the average duration of the real payouts—tend to be long. For example, when Britain was on the gold standard in the eighteenth and nineteenth centuries, nominal obligations were effectively real. At that time, the public debt was mainly long term ("funded") and often took the form of consols. The U.S. debt issued under the gold standard before World War I was also primarily long term; for example, most of the U.S. government bonds outstanding in 1916 had remaining maturities

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6 Alesina, Prati, and Tabellini (1990) and Giavazzi and Pagano (1990) argue on different grounds—to avoid confidence crises—that similar amounts of public debt should come due in each period.

7 Lucas and Stokey (1983) argue that consol debt may also be desirable on time-consistency grounds. In some situations, this maturity structure deters the government from making tax changes that would affect the term structure of real interest rates.

8 See Mitchell and Deane (1962, pp. 401-409).
in excess of 20 years.\textsuperscript{9}

Many developed countries have recently issued indexed bonds, and these securities tend to be long term. For example, the U.K. government has issued indexed coupon bonds with maturities as long as 38 years, which is nearly infinity. For other countries (as discussed in Bank of England [1995]), the issues of coupon bonds include Canada with 30-year maturity, Australia with 20 years, and Israel with 15 years. Sweden has issued \textit{discount bonds} with maturities of 19 years—the duration of a consol would be 19 years if the real discount rate were around 5-1/2%. The United States, which began the issue of indexed bonds only in 1997, began with a 10-year maturity. More generally, the observed short maturity for public debt in modern times applies mainly to nominal bonds in the context of a paper monetary standard.\textsuperscript{10} Nominal debt is considered in a later section.

Returning to equation (7), if the expected outlays differ across periods, then the debt structure no longer consists precisely of consols. A period with a high level of certainty-equivalent outlay has associated with it a correspondingly reduced level of debt coming due. For example, if a war or a major building project were anticipated for period $t$, then the debt would be structured at date 0 so that little debt would mature during period $t$. Otherwise, the government would have

\textsuperscript{9}See Board of Governors of the Federal Reserve System (1943, p. 411).

\textsuperscript{10}The usually stated maturity for nominal bonds overstates the duration not only because of the coupon payments but also because no account is taken of the diminished real value of future payouts due to inflation.
to borrow a lot in period $t$ at a financing cost that is uncertain at time 0. However, the practical relevance of this result is unclear, because it depends on the government having advance information about the future time pattern of public outlays.

If there were a positive drift in $G_t$—which has to be interpreted in the model as a drift in public spending as a ratio to GDP or some other measure of the tax base—then equation (7) calls for a negative trend in $B_{0t}$. Hence, the maturity structure of the debt would, in this case, be shorter term than a consol.

If the ratio of public outlay to the tax base had no drift but the levels of government spending and the tax base were each drifting upward—as would be expected with secular growth of the economy—then the optimal $B_{0t}$ would have a corresponding upward drift. Therefore, the maturity structure of the debt would be longer term than a consol in this situation.

2 A Limitation to non-Contingent Public Debt

A striking property of the previous solution is that the government arranges its debt obligations fully at date 0 and then never issues or buys back debt in subsequent periods. All the government does with respect to debt in future periods is, first, make contingent payments based on the realizations of the $G_t$ and, second, make the previously agreed non-contingent payouts (of roughly consol form).
These findings rely on the assumption that the government can use $G$-contingent debt effectively to convert its path of uncertain outlays into a deterministic path. The results would be different if the government were precluded from issuing $G$-contingent debt. However, the reason for this preclusion would likely not be on technical grounds involving the construction of the appropriate type of instrument. In countries with sophisticated financial markets and in which national-accounts data are available accurately without substantial delays, there would be no problem in creating this kind of debt contract.

The likely source of difficulty involves moral hazard—if debt payouts are contingent on the level of public outlay, then the government is likely to overspend, even perhaps to fight too many wars.\textsuperscript{11} In addition, the government might be

\textsuperscript{11}Such illustrious economists as Adam Smith and David Ricardo argued that governments have an excessive tendency to fight wars when the available method of war finance is too convenient. In their contexts, the overly convenient method of finance was viewed as debt issue, rather than current taxation, but the point about moral hazard is the same. Smith (1791, p. 427) said: "Were the expence of war to be defrayed always by a revenue raised within the year ... wars would in general be more speedily concluded and less wantonly undertaken. The people feeling, during the continuance of the war, the complete burden of it, would soon grow weary of it, and government, in order to humour them, would not be under the necessity of carrying it on longer than it was necessary to do so." Ricardo (1951, pp. 186-87) pointed out that wartime spending could be financed by taxes, borrowing, and borrowing with the establishment of a sinking fund and then said: "Of these three modes, we are decidedly of the opinion that the preference should be given to the first. ... When the pressure of war is felt at once, without mitigation, we shall be less disposed wantonly to engage in an expensive contest,
tempted to manipulate the statistics on spending to create the appearance of a contingency that warranted poor payouts on the debt. This problem might be acute because the relevant contingency involves not only the computation of the level of public outlays but also the scaling of these outlays in relation to some concept of a tax base.

Suppose then that the government is limited to non-contingent debt, taken here to be indexed bonds of various maturities. It would be technically straightforward to carry out the exercise of smoothing taxes as much as possible in the sense of the objective in equation (1) while limiting the government to the use of non-contingent debt. (See, for example, Giavazzi [1997] for a sketch of this exercise.) Then, instead of using indexed debt that was an approximation to a consol, the government would want to exploit any covariances between the future $G_t$ and the future prices of non-contingent debt, $P_{tj}$. For example, it is likely that a surprisingly high level of public outlay, $G_t$, would be associated with high riskless real interest rates and, hence, lower than expected values of the $P_{tj}$. Moreover, this effect tends to be greater at longer maturities, where asset prices are more sensitive to changes in real discount rates. (This effect depends also on the extent to which a current surprise in $G_t$ signals a long-term change in

and if engaged in it, we shall be sooner disposed to get out of it, unless it be a contest for some great national interest.” Clearly, Ricardo copied this idea from Smith, and it is therefore odd that Ricardo then went on to point out the economic equivalence of the three methods of paying for government spending (a point that Smith did not seem to recognize).
the level of public outlays.) The likely conclusion is that the government could
usefully hedge some of the uncertainty in the $G_t$ by tilting the maturity structure
toward more long-term debt and less short-term debt (or even toward the holding
of short-term assets). Hence, the optimal maturity structure would tend to be
even more long term than the consol structure derived before.

This kind of analysis would be valid if the rationale for the omission of $G$-
contingent debt were technical problems in setting up the right kinds of financial
contracts. In this case, it might be desirable to create the $G$-contingency indi-
directly by exploiting the covariances between $G_t$ and some other variable, such as
the $P_{ij}$, for which contingencies were feasible (in the case above by selecting the
maturity structure of the non-contingent debt).

However, the argument is invalid if—as seems plausible—the main reason for
the omission of the $G$-contingent debt involves moral-hazard problems of the sort
described earlier. In that case, the same moral-hazard problem arises when the
contingency on $G_t$ is attained indirectly. For example, if the maturity structure
of the non-contingent public debt were skewed toward the long end, and if an
increase in $G_t$ tends to depress the prices of long-term debt relative to short-
term debt, then the government would still have an excessive incentive to spend,
including to fight too many wars. (The government would, however, not have an
incentive to overstate the statistics on $G_t$, unless the asset prices reacted to the
stated values of the $G_t$, rather than the actual values.)
If the moral-hazard problem is so serious that it motivates the government to use explicitly $G$-contingent debt to a zero extent, then it seems that it would also motivate the government to use indirectly $G$-contingent debt to a zero extent. That is because the indirect contingency has the same moral-hazard problem but is otherwise less efficient than the direct method. For example, the indirect contingency achieved by skewing the maturity structure of non-contingent debt toward the long end has the problem of creating sensitivity of the government’s future financing costs to shifts in the $P_{ij}$ that are independent of the movements in the $G_t$. The avoidance of this sensitivity was the rationale for the consol financing in the first place.

Thus, my conjecture is that the full solution to the model with moral hazard—when this hazard is sufficient to preclude $G$-contingent debt issue—is that the government will also optimally avoid the exploitation of the covariance between the $G_t$ and the $P_{ij}$. To avoid this exploitation, the government will have to maintain the maturity structure of the indexed debt that was optimal when $G$-contingent debt was available, that is, the consol-type structure.

Even if the last conjecture is correct, the preclusion of $G$-contingent debt is important because it implies that the government will have to react to the realizations of the $G_t$ by altering the amount of debt outstanding. For example, a surprisingly high level of public outlay—as in a war—will be accompanied by the issue of new consol debt. In contrast, surprisingly low levels of outlay will
cause retirement of outstanding debt. This form of action describes pretty well the observed behavior of the British government over more than two centuries (see Barro [1987]).

3 Nominal Bonds

Suppose now that the government can issue nominal debt with varying maturities. Let $b_{0j}$ be the nominal amount committed in period 0 to be paid in period $j$ and $p_{0j}$ the associated time 0 real market price of these bonds. The real value of the future payouts depends on the realizations of future price levels. Future real prices of the nominal bonds, $p_{tj}$, depend on the price level for period $t$ and on the prospects at time $t$ for future inflation and real interest rates (which together determine nominal discount rates).

The probability distribution for inflation is treated here as exogenous to the government’s fiscal choices, and the distortions caused by inflation are assumed not to interact with those of other taxes. Bohn (1990) also takes this approach.

The stochastic properties of inflation are assumed to reflect some empirical regularities. One of these regularities, applicable to the paper monetary standards of modern times, is that innovations to inflation are highly persistent. In fact, the inflation rate is close to being non-stationary in post-World War II data, say for the United States and the United Kingdom.

Another apparent regularity is that innovations to inflation tend to signal bad