OPTIMAL MANAGEMENT OF Indexed AND Nominal DEBT

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Resumen
La composición óptima de la deuda pública respecto de su madurez y de su relación a estados de la naturaleza es analizado en relación al objetivo de suavización intertemporal de impuestos. El cumplimiento de dicho objetivo llevaría al gobierno a hacer depender sus pagos de deuda de los niveles de gasto público y de base impositiva. Si dichos niveles se alteran, pero los precios de activos de la deuda no-contingente son estocásticos, entonces para obtener plena suavización de impuestos existe una estructura de madurez óptima de la deuda no-contingente. Si los pagos de equivalencia cierta son los mismos para cada período, entonces el gobierno debiera garantizar pagos reales equivalentes en cada período, esto es, la deuda toma la forma de consolas indizadas. Esta estructura aisla el presupuesto del gobierno de variaciones no predecibles en los precios del mercado de bonos indizados de diversas madureces. Si no se puede emitir deuda contingente, entonces el gobierno puede querer desviarse de una estructura de madurez de consolas con el objeto de explotar covarianzas entre gasto público, la base tributaria y la estructura temporal de la tasa de interés real. Sin embargo, si la existencia de riesgo moral es la razón para ignorar la deuda contingente, entonces dicha consideración dificulta la explotación de las covarianzas y tiende a empujar la solución óptima a la estructura de madurez de consolas. La emisión de bonos nominales puede permitir al gobierno la explotación de covarianzas entre gasto público, la base tributaria y la tasa de inflación. Pero si lo que explica la ausencia de deuda contingente es la existencia de riesgo moral, entonces el mismo razonamiento hace indeseable la emisión de deuda nominal. La conclusión es que un enfoque de impuesto óptimo para analizar la posición de deuda pública, favorece el tener bonos indexados de largo plazo.

Abstract
A tax-smoothing objective is used to assess the optimal composition of public debt with respect to maturity and contingencies. This objective motivates the government to make its debt payouts contingent on the levels of public outlay and the tax base. If these contingencies are present, but asset prices of non-contingent indexed debt are stochastic, then full tax smoothing dictates an optimal maturity structure of the non-contingent debt. If the certainty-equivalent outlays are the same for each period, then the government should guarantee equal real payouts in each period, that is, the debt takes the form of indexed consols. This structure insulates the government’s budget constrain from unpredictable variations in the market prices of indexed bonds of various maturities. If contingent debt is precluded, then the government may want to depart from a consol maturity structure to exploit covariances among public outlay, the tax base, and the term structure of real interest rates. However, if moral hazard is the reason for the preclusion of contingent debt, then this consideration also deters exploitation of these covariances and tends to return the optimal solution to the consol maturity structure. The issue of nominal bonds may allow the government to exploit the covariances among public outlay, the tax base, and the rate of inflation. But if moral-hazard explains the absence of contingent debt, then the same reasoning tends to make nominal debt issue undesirable. The bottom line is that an optimal-tax approach to public debt favors bonds that are indexed and long term.

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In standard macroeconomics, fiscal policy involves choices about expenditures, taxes, and debt issue. The kinds of public spending may be distinguished with respect to their interactions with private decisions, for example, some public activities would influence private production and some would interact with households' choices of consumption and leisure. The taxes may also be differentiated by types; levies may fall on labor income, capital income, consumption, bodies, and so on.

The fiscal authority also chooses its type of debt obligations. These choices include the maturity structure of the debt, whether to issue nominal bonds or bonds indexed to the price level or a foreign currency, whether debt payments should be contingent on other variables, such as government expenditures and the state of the business cycle, and so on. These kinds of decisions are less familiar as a part of macroeconomics, although some aspects have been studied by Lucas and Stokey (1983); Persson and Svensson (1984); Bohn (1988, 1990); Calvo and Guidotti (1990); Alesina, Prati, and Tabellini (1990); Giavazzi and Pagano (1990); Chari, Christiano, and Kehoe (1994); Missale and Blanchard (1994); and Marce, Sargent, and Seppala (1996).

Optimal debt management can be thought of in three stages. First, if taxes are lump sum and the other conditions for Ricardian equivalence hold, as in Barro (1974), then the division of government financing between debt and taxes is irrelevant. Thus, the whole level of public debt will be indeterminate from an
optimal-tax standpoint.

Second, if taxes are distorting—for example, because the amount paid depends on an individual’s labor income or consumption—then the timing of taxes will generally matter, as in Barro (1979). This consideration tends to motivate smoothing of tax rates over time and thereby can make determinate the levels of debt at various dates. However, this element does not pin down the composition of debt, say by maturity.

Finally, if there is uncertainty about levels of public outlay, the tax base (say aggregate consumption or GDP), and asset prices, then the kinds of debt that the government issues will matter. In particular, the government may want to smooth tax rates over states of nature, and this consideration may dictate an optimal structure of the public debt. As one example, it may desirable for debt payouts to be conditioned on the level of government spending. As another example, it may be possible to design the maturity structure of the indexed debt so as to insulate the government’s financing costs from shifts in riskless real interest rates.

The strategy in this paper is to assume that the government desires to smooth the path of taxes when confronted by a path of exogenous, but stochastic, outlays. Other analyses, such as Zhu (1992) and Barro (1995), show that this objective can be derived, under some conditions, from the more fundamental objective of expected utility maximization for the representative household. The analysis assumes that policymakers can make effective commitments about the form of
future fiscal actions. Hence, unlike Lucas and Stokey (1983) and Persson and Svensson (1984), the debt composition is not set to ensure that policies are time consistent.

1 Public Finance with Tax Smoothing

The real public outlay for period $t$ is $G_t$. This outlay is exogenous and stochastic. The government sets its real tax revenue for period $t$ at the value $T_t$. The precise nature of the taxes is left unspecified. However, these levies are assumed to be distorting in such a way that the policymaker wishes to minimize the overall expected deadweight loss, as given from the perspective of an initial date, time 0, by

$$E_0 \sum_{j=1}^{\infty} w_j \cdot (T_{j+1} - T_j)^2$$  \hspace{1cm} (1)

where the $w_j > 0$ are weighting factors. The idea here is that variations in taxes over time cause distortions that the government would like to avoid. This objective will motivate smoothness in the $T_j$ across time and states.\(^1\)

If one think about levies on a tax base, such as income, consumption, or

\(^1\)The form of equation (1) is natural for consumption taxes in the absence of a labor-leisure choice. In that case, distortions reflect only variations in tax rates over time, not the levels of tax rates. With a labor-leisure choice, terms involving the levels of consumption or labor-income tax rates would also appear. The tax-smoothing behavior considered below is sometimes optimal in this extended setting. See Zhu (1992) for a general discussion of the optimality of tax smoothing.
property, then distortions are likely to increase more than in proportion with the amount of taxes when expressed in relation to the tax base. Therefore, $T_t$ and $G_t$ should be construed as ratios to the tax base. Uncertainty with regard to the tax base is analogous to uncertainty with respect to the level of public outlays, and a rise in $G_t$ can be viewed alternatively as an increase in government expenditure or a decrease in the tax base.

Indexed public debt issued at time $t$ pays the certain real amounts $B_{t1}, B_{t2}, ...$ in periods $t+1, t+2, ...$. These payouts can represent coupons or principal. The real market prices of this debt at time $t$ are $P_{t1}, P_{t2}, ...$. These asset prices are taken to be exogenous and stochastic, although the model could be extended to allow the choices of debt policy to affect the asset prices.

The government will also wish, in this model, to issue debt with payouts that are contingent on the realizations of the $G_t$. The amount of this debt issued at date 0 and due at date $t$ can be structured so that it pays off one unit less for each unit by which $G_t$ exceeds its date-zero expectation, $E_0G_t$. This debt can also be set up so that it pays a (positive or negative) non-contingent amount at date $t$, expressed as $\beta_{0t} \cdot E_0G_t$. This amount is assumed to be set so that the market value of contingent debt at date 0 is nil. That is, $\beta_{0t}$ is the premium (set at time zero) per unit of G-contingency. The amount payable in each period $t$ on

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2The debt therefore pays off badly when public outlays are surprisingly high or when the tax base is surprisingly low. The latter contingency is analogous to the GDP-linked bonds described by Shiller (1993).
the contingent debt is therefore

$$\beta_{0t} \cdot E_0 G_t + (E_0 G_t - G_t)$$

Since a high $G_t$ represents bad times—because high public outlay and a low tax base will typically be associated with low consumption—and the contingent debt pays off badly at these times, the premium $\beta_{0t}$ tends to be positive.

The government can achieve perfect tax smoothing in this model, that is, it can minimize the sum in equation (1) by attaining $T_1 = T_2 = \ldots = T$. First, the government issues the kind of $G$-contingent debt that has just been described. This issue effectively converts the path of uncertain outlays, $G_1, G_2, \ldots$ into a path of known outlays, $\hat{G}_1 = E_0 G_1 \cdot (1 + \beta_{01}), \hat{G}_2 = E_0 G_2 \cdot (1 + \beta_{02}), \ldots$ This contingent debt issue ensures that the government's tax smoothing will not be disturbed by surprises in the future levels of public outlays (and tax bases).

Second, the government has to manage its non-contingent debt to get the timing of taxes right, in other words, to ensure equal values of the $T_t$ even when the certainty-equivalent outlays, $\hat{G}_t$, vary over time. This problem would be simple if the future prices of non-contingent debt, $P_{tj}$, were known with certainty at date 0, that is, if riskless real interest rates were not subject to fluctuation. In that case, any maturity structure of the non-contingent debt—for example, one-period debt—could be used. The only concern, as in Barro (1979), would be to get the total quantity of debt issue correct in each period. However, this procedure does not work if the $P_{tj}$ are subject to uncertainty. In this case, unanticipated
shifts in these asset prices—and, hence, in the government’s refinancing costs—can affect the government’s budget constraint and thereby disturb the smoothing of taxes.

The quantities of non-contingent public debt of the various maturities at date 0 must satisfy the constraint:

\[
\sum_{j=1}^{\infty} B_{0j} P_{0j} = V_0
\]

where \( V_0 \) is the total market value of government debt (plus or minus) outstanding at date 0. This equation says that the government can rearrange its non-contingent debt as it wishes at the going market prices to achieve a desired distribution by maturity.

The government’s full outlay for period 1—including the non-contingent payout \( B_{01} \) established at date 0—is \( \hat{G}_1 + B_{01} \). This quantity is non-stochastic because the uncertainty in \( G_1 \) has been hedged by the issue of \( G \)-contingent debt. If taxes are successfully smoothed, then the revenue in each period is the same value, \( T \). If there is a gap between the full outlay and revenue in period 1, then the difference must be financed by non-contingent debt issue (plus or minus) at the prices, \( P_{ij} \), of non-contingent debt then prevailing. However, if each of these asset prices contains an independent random element, then any debt issues of this type will cause tax smoothing to fail, because the realizations of the asset prices will impact on required levels of future taxes.\(^3\) Hence, full tax smoothing

\(^{3}\)The assumption here is that Ponzi games are precluded and, hence, an effect on the gov-
requires a balance between full outlay and revenue in period 1:

$$\dot{G}_1 + B_{01} = T$$  \hspace{1cm} (3)

Since no new debt is issued and no old debt is retired in period 1, the full outlay for period 2 is \( \dot{G}_2 + B_{02} \). The same reasoning as that applied to period 1 implies that this outlay for period 2 must equal the tax revenue, \( T \). Proceeding forward in time, the conclusion is that the form of equation (3) must hold for every period \( t \):

$$\dot{G}_t + B_{0t} = T$$  \hspace{1cm} (4)

Multiplication of both sides of equation (4) by \( P_{0t} \) and summation from \( t = 1 \) to \( \infty \) leads, after substitution from equation (2), to a formula for \( T \):

$$T = \frac{V_0 + \sum_{j=1}^{\infty} P_{0j} \dot{G}_j}{\sum_{j=1}^{\infty} P_{0j}}$$  \hspace{1cm} (5)

This result says that the constant flow of real taxes in each period equals the permanent flow of spending, which includes the required financing on the initial debt, \( V_0 \), plus the permanent flow of outlay. The last quantity weighs each amount \( \dot{G}_j \) by the present-value factor, \( P_{0j} \). For example, if the one-period, non-contingent real interest rate were the constant \( r \), so that \( P_{0j} = 1/(1 + r)^j \), then

$$T = r \cdot [V_0 + \sum_{j=1}^{\infty} \frac{1}{1 + r}^j \dot{G}_j]$$  \hspace{1cm} (6)

A government’s budget in any period must, for given public outlays, show up eventually in taxes.