we capture the volatility in the elasticities of the labor supply, increasing the standard deviation of productivity and hours worked. Also, in particular, the model's volatility of inflation and interest rates exceeds that of the data (by a factor of two), while the volatility of velocity is lower than the actual value (the model explains almost 50% of the actual volatility). For most of the real variables it is hard to reject the null hypothesis that the model predicts individual accurate moments.

In implementing the three joint tests we can easily see that the model which includes the fiscal sector performs better than the model with no fiscal sector. The evaluation of the overall model with the $\chi^2(10)$ joint test, indicates that, including all real and nominal variables from the table (excluding CPI), the model has good performance and it is not rejected with respect to the actual U.S. values. When we consider only the real variables, the model without the fiscal sector is not rejected with a 5% significance level, however the performance of the model is notably improved once we include the fiscal policy. Now the model presents a p-value range from 0.15 to 0.20. From the point of view of this paper, the most important test is the first joint test $\chi^2(4)$. It represents the joint test considering the last four variables from tables 2a-c. This test presents the stronger favorable results with respect to the model with taxes, with p-values around 0.50. When the CRRA parameter is 0.5 the model with or without fiscal sector predicts almost the same results in terms of the tests: the model is not rejected at 5%. Once the CRRA moves to 1.0, the p-value reaches 0.509 for the model without taxes. Including taxes improves the model performance; now the p-value is 0.543. Finally, in the last simulation, considering a CRRA parameter of 1.5, the same modification increases the p-
value from 0.495 to 0.553. In summary, the performance of the model with respect to inflation, velocity, interest rate, and deficit improves once we consider a stochastic fiscal policy.

Now, we analyze the performance of the model with respect to the cross correlations. In Tables 3.a and 3.b are presented the simulated moment with their respective $\chi^2$ statistics. The correlation of consumption and output is well captured by almost all the simulated economies. For the first economy with CRRA of 0.5, the p-value is equal to 0.01 for the economy without taxes; once we include taxes, this value increase to 0.09. In the second economy, with a 1.0 CRRA parameter, the p-values change from 0.18 to 0.20, again showing the good performance of the model. This results is repeated when $\gamma = 1.5$. The same analysis follows for the price level, except that the inclusion of the stochastic taxes changes the sign of the correlation coefficient when the CRRA is greater that 1.0. The level of the correlation between output, and either consumption, investment, or hours is well captured by the model.

Even when the individual tests do not show a good performance for the model, this is because the variability of the correlations is very low. However, considering only the level of the correlations we can see that in general, the model explains very well the actual correlations.

In summary, the percentage of explanation in volatility of consumption is almost 90%, investment 80%, capital stock 82%, hours 72%, productivity 60%, price level 120%, velocity 50%, interest rate 300%, and deficit 75%. Hence, looking at the joint tests the model with stochastic fiscal policy show a marked improvement in terms of the nominal variables and in term of the real variables.
In Figures 3 and 4 are represented three scatter plots. The first plot in Figure 3 shows the actual data relation between the fiscal deficit and the interest rate, fitting an OLS curve through the points. The next two plots in the same Figure shows the simulated relation for these two variables: one considering a CRRA parameter of 0.5, while in the other is 1.5, both assuming an economy with stochastic fiscal policy. Here we can see the relatively high volatility of the actual versus simulated data for the deficit, and the low volatility of the actual interest rate. The Figure presents the OLS curve estimated with those data, and we are able to see that our model captures very well the sign in the cross relation between these two variables.

Figure 4 presents the same kind of analysis but now we include velocity instead of deficit. We can see the positive relation between velocity and interest rate, and the relatively high volatility in the actual data in comparison with the two model simulations. Although the sign of the cross relation is well captured by the model, it seems to be a little underestimated.

3.3. VAR Analysis: Impulse Response Functions.

In this section we estimate a statistically well specified vector autoregressive (VAR) model, considering a subset of five real and nominal variables: interest rate, money, consumption, velocity and deficit. The first subsection examines the stationarity of the data, using the classical unit roots literature. Following the implementation of the unit root tests, we analyze the existence of cointegration among the variables, to establish the necessity of error correction mechanisms inside the estimated VAR. Finally we implement Granger-Causality tests to establish the order of the VAR.
3.3.1. *Unit Roots and Cointegration.*

The first step in estimating a VAR is to determine the existence of an integrated process in the variables of interest\textsuperscript{26}. Tests of unit roots are designed to establish such possibility. In Table 4 we present the Augmented Dickey-Fuller (ADF) unit root tests for a subset of real and nominal variables. The ADF equation was specified considering one, two and four lags, to ensure that the error term is a white noise. For each of the specifications was considered the existence of a constant and a trend variable in the equations. Columns (1) denoted the pure ADF test, without trend and constant term; columns (2) include the constant term, while columns (3) is a complete ADF with a constant and a trend term. The shaded cell means that the null hypothesis of no unit root is rejected with 5% of significance level.

The results are pretty standard and are consistent with the literature. Treham and Walsh (1990) evaluate the existence of nonstationarity in government expenditure, tax rate, inflation and velocity, while Stock and Watson (1993) have presented evidence that output, real balances, and nominal interest rates are integrated of order one and cointegrated\textsuperscript{27}. Price level, interest rate, real money, inflation and velocity are among the candidates for high probability of not rejecting the unit root null. For money, output, consumption and deficit, the tests still do not reject the null of a unit root, but not as strongly as the other variables just mentioned. The results are

\textsuperscript{26}A process that is integrated of order p, or I(p), must be differenced p times to be stationary. See Hamilton (1994) or Granger and Newbold (1986).

\textsuperscript{27}Ahmed and Rogers (1995), using a long annual data set, employed Phillips-Perron (1988) and Perron (1989) tests for unit root in some fiscal and real variables for the U.S. and U.K. The statistics reported indicate a failure to reject the unit root null for the levels of each of the variables, but rejection for the first differences.
consistent when we consider three lags and, at the same time, for the different specifications of the ADF tests.

Stock and Watson (1993) reported some degree of cointegration among output, real balances, and interest rates. Using annual data from 1900-1989, and looking for stability in the money demand equation, they found that the residuals constructed using either the full-sample or first-half point (1900-1945) estimates are consistent with cointegration, while the residuals based on the postwar estimates are not. Our results are consistent with their results, not rejecting the null of no cointegration. Table 5 reports these results. The entries in this table report the test of cointegration between any pair of variables listed in the first column and the first row. Using a ADF test with constant, trend and four lags\(^{28}\), we never reject with a 5\% of significance level the null of unit root in the residuals of the cointegrated equation. Particularly important for the public finance literature are the results with respect to deficit. The last second row of Table 5 (before the comments) reports the cointegration between deficit and the rest of the variables under consideration. It seems that the data report some degree of long term relationship between deficit and either interest rate or inflation since the p-values are around 0.10. Even when we did find strong evidence to reject the null of no cointegration (with a 10\% and with ADF tests), the entries give us some idea that some pairs are actually cointegrated, i.e., that there is a good possibility that in the long run some of the process variables are really linked. This is the case for the pairs money-inflation, velocity-inflation, and, real balances-output, among many others.

\(^{28}\)The results are robust to the ADF test specification equation.
3.3.2. *Dimension of the Model: AIC, HQ and BIC Tests.*

Given that we did not reject statistically the null of no cointegration and unit root in the residuals (as Stock and Watson (1993)), it is not necessary to revise the specification of the VAR to introduce any error correction terms in the data generation process. The only thing left is the specification of the order for the variables in the Choleski decomposition. Because we compare the simulated HP-filtered data, it was necessary to recalculate the unit root tests for the actual data after being filtered by the Hodrick-Prescott filter. The results are a quite different in comparison with the unit root tests for the data without detrending. In Table 6 we report the ADF tests for the elected subset of variables (this table follows the same structure than Table 4). Here, and for most of the specifications, either interest rate or deficit do not present evidence of having a unit root in the process, changing the results found in Table 4. However, for the other three variables (money, consumption, and velocity) the unit root tests confirm the null of a unit root. Having these results in mind, the VAR will include two variables in levels (interest rate, and deficit) and three in first differences (money, consumption, and velocity).

Once we determine the included variables, it is necessary to specify the order to do the decomposition for the impulse response analysis. To study causality among the variables, we implement the Granger-causation test (Granger (1969)); the results are reported in Table 7. Entries off the diagonal indicate the value of the test under the null $H_0$: $X_1$ is not Granger-caused by $X_2$, with its respective p-values in parentheses below the test. This test can be

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\(^{29}\) All the variables are generated from previously log-HP-filtered data.

\(^{30}\) For a review of the VAR estimation procedure see Sims (1980).
done with different specifications of the causality equation, basically to ensure white noise in the residuals. We implement the F tests with different number of lags, all with the same causality results. Table 7 reproduces the results with 1 lag for the Granger equation.

The test values, altogether with the p-values, indicates that there is a causality going from interest rate to money growth (we reject the null that money is not caused by interest rate with test equal to 16.5, and a p-value 0.0001). The same is true for interest rates with consumption growth and velocity growth. Both hypotheses are rejected with a p-value lower that 0.01 (1%). Other important results are that velocity growth causes money growth (p-value 0.0016) and consumption (p-value 0.07), and that consumption growth causes the deficit (p-value 0.0006). From these results, we conclude that the final ordering will be: interest rate, velocity growth, money growth, consumption growth, and, deficit (r-dv-dm-dc-def).

Finally, it is necessary to define the correct dimension of the model. There are many tests that address that question. However, based on Lutkepohl 1985's Monte Carlo simulations, among the most robust tests are the Akaike Information Criterion (Akaike (1974)), the Hannan and Quinn Criterion (Hannan and Quinn (1979)), and the Bayesian or Schwarz Information Criterion (Schwarz (1978))\textsuperscript{31}. What these criteria do is minimize the following function,

\[
\hat{\lambda}_{(k)} = \ln |\hat{\Sigma}_k| + \left( \frac{k \cdot d^2}{T} \right) \cdot \Delta
\]

\textsuperscript{31}Here and after AIC, HQ and BIC.
where \( \ln|\hat{\Sigma}_x| \) represent the logarithm of the determinant of the variance-covariance matrix for the residuals in the equation with "k" lags (one up to six). The number of equations in the VAR is represented by "d" (five in our problem), and the total number of observations is denoted by "T".

The specific representation for \( \Delta \) depends on the criteria used. For the Akaike criterion \( \Delta = 2 \), Hannan and Quinn use \( \Delta = 2 \cdot \ln(\ln(T)) \), while in the Schwarz criterion \( \Delta = \ln(T) \). Table 8 reproduces these results for a span of lags from one up to six.

As we expect, when the number of lags increase, the value of the logarithm of the determinant decreases, which means that we are always going to choose the maximum number of lags based on traditional Tiao-Box-Sims criterion (Sims (1980) and Tiao and Box (1981)). The existence of the AIC, HQ and BIC criteria solve this problem, and more importantly, based on Monte Carlo simulations, we know the relative power of the tests. Once we consider different criteria (AIC,HQ,BIC) the \( \lambda_{(k)} \) function chooses three lags or one lag (AIC, and HQ and BIC respectively), given the same results when the specification of the VAR is made in terms of levels instead of first differences for money, consumption and velocity. This gave us confidence in choosing 1 as the optimal number of lags to define the dimension of the model. Now we go to analysis of the impulse response functions.

3.3.3. Impulse Response and Variance Decomposition.

Based on the VAR previously specified, we estimate the impulse response functions for interest rate, velocity and deficit. First, we see in Figure 5 the response of the variables in the model to a one standard deviation shock in the level of interest rate. The first plot of this figure reports the impulse
response functions based on the actual data, while the second two graphs present the responses of the models with fiscal sector and CRRA parameter 0.5 and 1.5, respectively. A common feature of the calibrated models is the lack of persistence, however this is not the case in our model. From the impulse response functions showed in the second and third graph in Figure 5 we can see that both the actual and simulated data, show a persistence that lasts for about seven quarters. Moreover, the sequence of deficits seems to be well represented by the model. In the actual data, the shock to interest rates increase the level of deficit for about three quarters, but also there is an increase in velocity that lasts 7 quarters. The responses of deficit and velocity (in less percentage) are captured by the model. The sequence of money, however, goes in the wrong direction.

In Figure 6 we can see the response of interest rate to a one standard deviation shock in all the VAR variables. Now the model loses a little persistence relative to the actual data impulse response functions. While most of the variables induce a 7 to 8 response in the interest rate, the model only generates 5 quarters of response, half a year less. However, the signs of the interest rate process are very similar in both the actual and simulated models. A one standard deviation shock in deficit induces an increase in interest rate that lasts 2 years in the actual data, and one year in the simulated model. The shock in velocity growth generates a 6 quarters negative response in the actual data and 3 quarters for the model. The model seems to reproduce with success the behavior of interest rates as the economy encounters an unanticipated shock.

The same analysis performed with interest rates in the last figure is