The Leverage Cycle, Default, and Foreclosure

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At least since the time of Irving Fisher, economists, as well as the general public, have regarded the interest rate as the most important variable in the economy. But in times of crisis, collateral rates (margins or leverage equivalently) are far more important. Despite the cries of newspapers to lower the interest rates, the Fed would sometimes do much better to attend to the economy-wide leverage and leave the interest rate alone.

When a homeowner (or hedge fund or a big investment bank) takes out a loan using, say, a house as collateral, he must negotiate not just the interest rate, but how much he can borrow. If the house costs \$100 and he borrows \$80 and pays \$20 in cash, we say that the margin, or *haircut* is 20%, the loan to value is \$80/\$100 = 80%, and the collateral rate is \$100/\$80 = 125%. The leverage is the reciprocal of the margin, namely, the ratio of the asset value to the cash needed to purchase it, or \$100/\$20 = 5. These *ratios* are all synonomous.

In standard economic theory, the equilibrium of supply and demand determines the interest rate on loans. It would seem impossible that one equation could determine two variables, the interest rate and the margin. But in my theory, supply and demand do determine both the equilibrium leverage (or margin) and the interest rate.

It is apparent from everyday life that the laws of supply and demand can determine both the interest rate and leverage of a loan: the more impatient borrowers are, the higher the interest rate; the more nervous the lenders become, or the riskier the asset prices become, the higher the collateral they demand. But standard economic theory fails to properly capture these effects, struggling to see how a single supply-equals-demand equation for a loan could

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determine two variables: the interest rate and the leverage. The theory typically ignores the possibility of default (and thus the need for collateral), or else it fixes the leverage as a constant, allowing the equation to predict the interest rate.

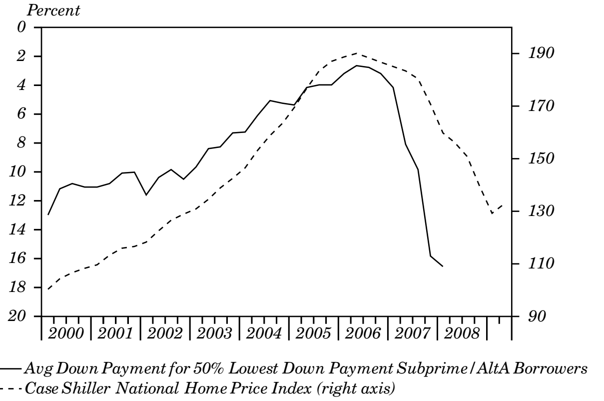
Yet, variation in leverage has a huge impact on the price of assets, contributing to economic bubbles and busts. This is because for many assets there is a class of buyers for whom the asset is more valuable than it is for the rest of the public (standard economic theory, in contrast, assumes that asset prices reflect some fundamental value). These buyers are willing to pay more, perhaps because they are more optimistic, or they are more risk tolerant, or they simply like the assets more, or they are important hedges for them and not for the others. If they can get their hands on more money through more highly leveraged borrowing (that is, getting a loan with less collateral), they will spend it on the assets and drive those prices up. If they lose wealth, or lose the ability to borrow, they will buy less, so the asset will fall into more pessimistic hands and be valued less.

In the absence of intervention, leverage becomes too high in times when markets have been stable and apparently devoid of risk for long periods of time, and too low in scary times when asset prices are very uncertain. The high leverage during the safe period makes the economy much more vulnerable when uncertainty returns. As a result, in boom times asset prices are too high, and in crisis times they are too low. This is the leverage cycle.

Leverage dramatically increased in the United States and globally from 1999 to 2006. A bank that in 2006 wanted to buy a AAA-rated mortgage security could borrow 98.4% of the purchase price, using the security as collateral, and pay only 1.6% in cash. The leverage was thus 100 to 1.6, or about 60 to 1. The average leverage in 2006 across all of the US$2.5 trillion of so-called 'toxic' mortgage securities was about 16 to 1, meaning that the buyers paid down only $150 billion and borrowed the other $2.35 trillion. Home buyers could get a mortgage leveraged 35 to 1, with less than a 3% down payment. Security and house prices soared.

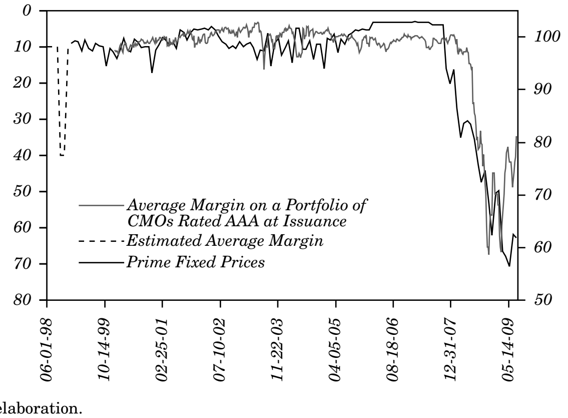
By 2009 leverage had been drastically curtailed by nervous lenders wanting more collateral for every dollar loaned. Those toxic mortgage securities were leveraged on average only about 1.2 to 1. A homeowner who bought his house in 2006 by taking out a subprime mortgage with only 3% down could not take out a similar loan in 2009 without putting down 30% (unless he qualified for one of the government rescue programs). The odds are great that he wouldn't

Figure 1. Securities Leverage Cycle, Margins Offered and AAA Securities Prices



Source: Author's elaboration.
The chart represents the average margin required by dealers on a hypothetical portfolio of bonds subject to certain adjustments noted below. Observe that the Margin % axis has been reversed, since lower margins are correlated with higher prices.The portfolio evolved over time, and changes in average margin reflect changes in composition as well as changes in margins of particular securities. In the period following Aug. 2008, a substantial part of the increase in margins is due to bonds that could no longer be used as collateral after being downgraded, or for other reasons, and hence count as 100% margin.

Figure 2. Housing Leverage Cycle, Margins Offered (Down Payments Required) and Housing Prices

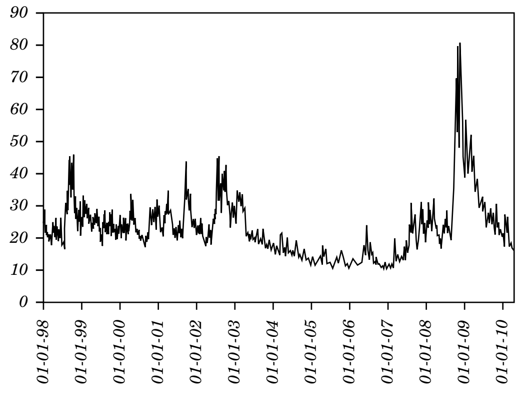


Source: Author's elaboration.
Observe that the Down Payment axis has been reversed, because lower down payment requirements are correlated with higher home prices. For every AltA or Subprime first loan originated from Q1 2000 to Q1 2008, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and 2.7% down payment in Q2 2006 corresponds to leverage of about 37. Subprime/AltA issuance stopped in Q1 2008.

have the cash to do it, and reducing the interest rate by 1 or 2% wouldn't change his ability to act.

Seven and a half years after the crash of subprime mortgages in February 2007, the economy still has not returned to normal. The Fed has lowered interest rates to near 0 and kept them there for five years. But it has not tried to boost leverage, except for a brief successful period in 2009 and 2010.

Figure 3. VIX Index



Source: Author's elaboration.

The leverage cycle is a recurring phenomenon. The financial derivatives crisis in 1994 that bankrupted Orange County in California was the tail end of a leverage cycle. So was the emerging markets mortgage crisis of 1998, which brought the Connecticut-based hedge fund Long-Term Capital Management to its knees, prompting an emergency rescue by other financial institutions. The crash of 1987 also seems to be at the tail end of a leverage cycle. The Tulip Bulb mania and the Japanese land boom of the 1980s were leverage cycles.

The policy implication of my theory of equilibrium leverage is that the Fed should manage system wide leverage, curtailing leverage in normal or ebullient times, and propping up leverage in anxious times. The theory challenges the "fundamental value" theory of asset pricing and the efficient markets hypothesis.

If agents extrapolate blindly, assuming from past rising prices that they can safely set very small margin requirements, or that falling prices means that it is necessary to demand absurd collateral levels, then the cycle will get much worse. But a crucial part of my leverage cycle story is that every agent is acting perfectly rationally from his own individual point of view. People are not deceived into following illusory trends. They do not ignore danger signs. They do not panic. They look forward, not backward. But under certain circumstances the cycle spirals into a crash anyway. The lesson is that even if people remember this leverage cycle, there will be more leverage cycles in the future, unless the Fed acts to stop them.

The leverage cycle always involves the same elements. First, a sustained period of calm leads lenders to increase loan to value ratios, both because they feel safe and because financial innovation is given time to further stretch collateral. This leads to higher asset prices as more people can afford the downpayment to buy more assets or with indivisible assets, to buy the asset at all. Borrowing thus goes up for a squared reason: a higher percentage is borrowed of higher valued assets. Next a little bit of bad news occurs. This causes prices to drop a little, which in turn leads to huge losses for the most optimistic, leveraged buyers. The redistribution of wealth from optimists to pessimists further erodes prices, causing more losses for optimists. If lenders gauge future uncertainty by extrapolating from the past, then these price declines make them nervous and cause them to set tighter margins. Alternatively, even if they rationally forecast the future, and the news is not just bad, but scary, in the sense that it increases uncertainty, they will also tighten margins. This leads to steeper price declines, which causes leveraged optimists to lose more money, which causes rational lenders to anticipate further price declines, leading then demanding more collateral, and so on. All three elements feed back on each other.

The best way to stop a crash is to act long before it occurs, by restricting leverage in ebullient times. The best time for an investor to enter the market is just after the crash.

My theory is of course not completely original. Over 400 years ago in the Merchant of Venice, Shakespeare explained that to take out a loan, one had to negotiate both the interest rate and the collateral level. It is clear which of the two Shakespeare thought was the more important. Who can remember the interest rate Shylock charged Antonio? (It was zero percent.) But everybody remembers the pound of flesh that Shylock and Antonio agreed on as collateral. The upshot of the play, moreover, is that the regulatory authority (the court) decides that the collateral Shylock and Antonio freely agreed upon was socially suboptimal, and the court decreed a different collateral: a pound of flesh but not a drop of blood. The Fed should also decree different collateral rates sometimes.

In more recent times there has been pioneering work on collateral by Shleifer and Vishny SV (1992), Bernanke, Gertler, Gilchrist BGG (1996, 1999), and Holmström and Tirole (1997). This work emphasized the asymmetric information between borrower and lender, leading to a principal agent problem. In Holmström and Tirole (1997) the managers of a firm are not able to borrow all the inputs necessary to build a project because lenders would like to see them put skin in the game, by putting their own money down, to guarantee that they exert maximal effort. The BGG (1999) model, adapted from their earlier work, is cast in an environment with costly state verification. I do not invoke any asymmetric information. I believe that it is important to note that endogenous leverage need not be based on asymmetric information. Of course the asymmetric information revolution in economics was a tremendous advance, and asymmetric information plays a critical role in many lender-borrower relationships; however, sometimes the profession becomes obsessed with it. In the crisis of 2007 – 2009, it does not appear to me that asymmetric information played a critical role in setting margins. Certainly the buyers of mortgage securities did not control their payoffs. In my model the only thing backing the loan is the physical collateral. Because the loans are no-recourse, there is no need to learn anything about the borrower. All that matters is the collateral. Repo loans, and mortgages in many states, are literally no-recourse. In the rest of the states, lenders rarely come after borrowers for more money beyond taking the house. And for subprime borrowers, the hit to the credit rating is becoming less and less tangible. In looking for determinants of (changes in) leverage, one should start with the distribution of collateral payoffs, and not the level of asymmetric information.

Another important paper on collateral is Kiyotaki and Moore (1997). Like BGG (1996), this paper emphasized the feedback from the fall in collateral prices to a fall in borrowing capacity, such as would occur from a constant loan to value ratio. By contrast, my work defining collateral equilibrium focused on what determines the *ratios* (LTV, margin, or leverage) and why they change. In practice, I believe the change in ratios has been far bigger and more important for borrowing than the change in price levels. The possibility of changing ratios is latent in the BGG models, but not emphasized by them. In my 1997 paper I showed how one supply-equals-demand equation can determine leverage as well as interest even when the future is uncertain. In my 2003 paper on the anatomy of crashes and margins (it was an invited address at the 2000 World Econometric Society meetings), I argued that in normal times leverage and asset prices get too high, and in bad times, when the future looks worse and more uncertain, leverage and asset prices fall too low. In the certainty model of Kiyotaki and Moore, to the extent leverage changes at all, it goes in the opposite direction, getting looser after bad news. In Fostel and Geanakoplos (2008b), on leverage cycles and the anxious economy, we noted that margins do not move in lockstep across asset classes, and that a leverage cycle in one asset class might spread to other unrelated asset classes. In Geanakoplos and Zame (2009, 2014) we describe the general properties of collateral equilibrium. In Geanakoplos and Kubler (2005), we show that managing collateral levels can lead to Pareto improvements.[1]

The recent crisis has stimulated a new generation of important papers on leverage and the economy. Notable among these are Brunnermeier and Pedersen (2009), anticipated partly by Gromb and Vayanos (2002), and Adrian and Shin (2009), and Simsek (2013).

This paper emphasizes two dangers to leverage. The first is that the roller coaster of leverage, caused by changes in risk perceptions, leads to a roller coaster in asset prices. That has all sorts of implications for the risk exposure of agents who are forced to hold these assets and cannot hedge them (like households who own houses or banks whose major business is holding mortgages). Second, when a boom is followed by a bust, many borrower will find themselves under water, owing more than the value of the collateral. There are typically large losses in turning over the collateral, partly because of vandalism and so on, and partly because agents have no incentive to invest in their collateral when they know it may be seized anyway. Subprime lenders (bondholders) received on average less than 25% of the loan amount back when they foreclosed on a home during the years 2007–12. We shall see that in the model even though every lender rationally anticipates the incentives his borrowers will face, they still collectively extend too much leverage because no lender takes into account that if he reduced his loan size the price of housing would be slightly higher in the future and some other homeowner might not go underwater and stop fixing his house.

1. For Pareto improving interventions in credit markets, see also Gromb-Vayanos (2002) and Lorenzoni (2008).

Section 2 describes a very simple two period model of collateral equilibrium with one risky asset. This enables me to introduce the notation gently and to display the connection between uncertainty, leverage, and asset prices in graphical form. There it is explained why the limits to borrowing that arise when collateral is needed to guarantee delivery can paradoxically increase the price of assets that need to be purchased with borrowed funds. In section 3, I introduce general notation for collateral equilibrium. Then I describe the leverage cycle. In section 4, I introduce delays in unencumbering collateral and the resulting costs of foreclosure. This brings out one of the negative externalities caused by increased leverage.

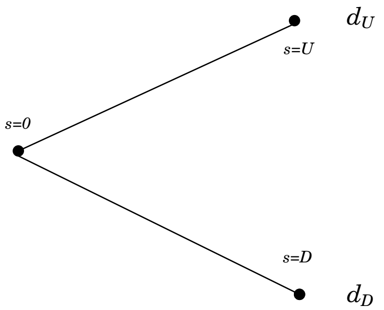
1. A Two-Period, Binomial Economy with One Risky Asset

To introduce our notation and to illustrate some of the analytical ideas in a simple environment, let us consider the following family of examples taken from Geanakoplos (2003). For this family of examples we define equilibrium without financial contracts, Arrow Debreu equilibrium, and collateral equilibrium. We end by comparing asset prices across the different equilibria.

Consider two time periods 0,1 , and two states of nature U and D in the last period and agents or households $h \in H$. Suppose that there are three commodities at time 0, whose holdings are denoted by $x_0 = (x_{01}, x_{02}, x_{03}) = (c_0, y_0, w_0)$ which we call the perishable consumption good C, the durable asset Y, and the durable ("warehousable") consumption good W. Suppose there is just one commodity in each state U and D, which we think of as the perishable consumption good, and whose holdings we denote by $x_s = c_s$, $s = U, D$. We think of the durable consumption good as something like cigarettes or canned food or oil in a well, that can be stored costlessly until the next period, or costlessly transformed one to one into the consumption good and used up immediately, by lighting the cigarette or opening the can of sardines, or drawing the oil out of the well.

Each unit of Y pays either d_U or $d_D < d_U$ of the consumption good in the two states U (as in Up) or D (as in Down), respectively. Imagine the asset as a mortgage that either pays in full or defaults with recovery d_D. (All mortgages will either default together or pay off together). But it could also be an undrilled oil well that could be a gusher or a small one. The only difference between W and Y is that the output of W is known for sure to be 1 next period, while the output of Y is uncertain.

Figure 4. Simple Binomial Tree



Source: Author's elaboration.

Let us assume that every agent h has a continuous, concave and monotonic von Neumann Morgentern utility u^h for the perishable consumption good in each state, discount factor δ_h, and probability belief γ_U^h for state U and probability belief $\gamma_D^h = 1 - \gamma_U^h$ for the down state D. The agents are characterized by their utilities and endowments

$$U^h(c_0, y_0, w_0, c_U, c_D) = u^h(c_0) + \delta_h[\gamma_U^h u^h(c_U) + \gamma_D^h u^h(c_U)]$$

$$e^h = (e^h_{C_o}, e^h_{Y_o}, e^h_{W_o}, e^h_{C_U}, e^h_{C_D})$$

The durable consumption good and the asset provide no direct utility to their holders at time 0, they just increase income in the future. Moreover their future value does not depend on who holds them at time 0. We call such assets financial assets, in contrast to houses, that do provide immediate utility at time 0 to those who hold them.

To complete the formal description of our example, we must also specify the production technology. We let the matrices

$$E_U = [0 \;\; d_U \;\; 1], E_D = [0 \;\; d_D \;\; 1]$$

denote what happens next period to each of the commodities at time 0. The first column of each matrix corresponds to the dividend in states U and D of holding the perishable consumption good at time 0. The second column corresponds to the dividend in states U and D of holding the durable asset Y, and the third column corresponds to holding holding the durable consumption good W ("warehousing" or "storing" it). Thus an agent who holds $x_0 = (x_{01}, x_{02}, x_{03}) = (c_0, y_0, w_0)$ in period 0 receives E_U, x_0 of dividends at U and E_D, x_0 of dividends at D.

We also describe the intraperiod technology

$$Z_0 = \{z = (z_{01}, z_{02}, z_{03}) : z \leq (\lambda, 0, -\lambda), \lambda \in \mathbb{R}\}$$

which respresents the idea that the durable consumption good can be transformed one to one into the perishable consumption good and vice versa. We suppose every agent has access to this technology.

1.1 A Continuum of Risk Neutral Agents and the Marginal Buyer

Let us consider the simplest possible agents. Suppose the agents $h \in H$ only care about the total expected consumption they get, no matter when they get it. They are not impatient.

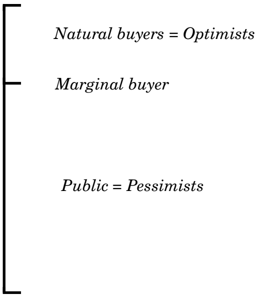
Thus $\delta_h = 1$ and $u^h(c) = c$ for all $h \in H$. The difference between the agents is thus only in the probabilities $\gamma_U^h, \gamma_D^h = 1 - \gamma_U^h$ each attaches to the good outcome of Y and the bad outcome. We suppose that γ_U^h is strictly monotonically increasing and continuous in h so that the higher h is, the more optimistic is the agent. When H is a finite set, the continuity hypothesis is vacuous. But we consider the case where H is the unit interval with the uniform Lebesgue measure. For this continuum case, the summation over $h \in H$ must always be understood as the integral over $H = [0,1]$ with respect to the standard Lebesgue measure.

The advantage of the continuum of agents approach is that every agent will always be able to optimize by going to one extreme or another, for example putting all its wealth into the risky asset Y or into the riskless asset W. But one agent, which we shall call the marginal buyer, will be indifferent to both extremes.

1.2 Equilibrium Asset Pricing without Credit

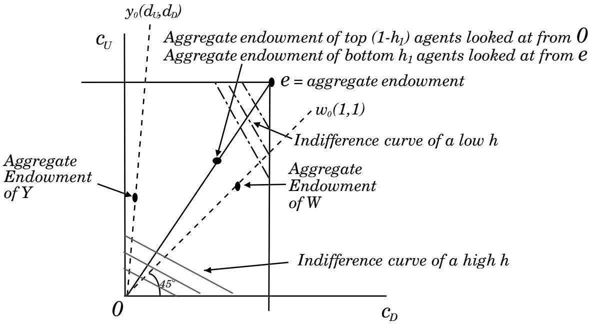
We can always choose the perishable consumption good as the numeraire in every state 0,1 and 2; hence we take its price to be 1 in every state. Since the storable consumption good is transformable

Figure 5. Marginal Buyer Theory of Price



Source: Author's elaboration.

Figure 6. Edgeworth Box with a Continuum of Risk Neutral Agents



Source: Author's elaboration.

one to one into the perishable consumption good, we can also take the price of W_0 to be 1. Suppose the price of the asset per unit at time 0 is p_Y , somewhere between 0 and 1.

If borrowing were not allowed, and agents could only trade the commodities among themselves in period 0, then the budget set for each agent would be

$$B_0^h(p) = \{(c_0, y_0, w_0, c_U, c_D) \in \mathbb{R}_+^5 : c_0 + w_0 + p_Y(y_0 - e_{Y_o}^h) = e_{C_o}^h + e_{W_o}^h$$

$$c_U = 1w_0 + d_U y_0 + e_{C_U}^h$$

$$c_D = 1w_0 + d_D y_0 + e_{C_D}^h\}.$$

Given the price p_Y, each agent chooses the consumption plan $(c_0^h, y_0^h, w_0^h, c_1^h, c_2^h)$ in $B_0^h(p_Y)$that maximizes his utility U^h defined above. In equilibrium all markets must clear

$$\sum_{h\in H}(c_0^h + w_0^h) = \sum_{h\in H}(e_{C_o}^h + e_{W_o}^h)$$

$$\sum_{h\in H} y_0^h = \sum_{h\in H} e_{Y_o}^h$$

$$\sum_{h\in H} c_U^h = d_U \sum_{h\in H} e_{Y_o}^h + 1 \sum_{h\in H} w_0^h + \sum_{h\in H} e_{C_U}^h$$

$$\sum_{h\in H} c_D^h = d_D \sum_{h\in H} e_{Y_o}^h + 1 \sum_{h\in H} w_0^h + \sum_{h\in H} e_{C_D}^h$$

The agents h who believe that

$$\gamma_U^h d_U + (1 - \gamma_U^h) d_D > p_Y$$

will spend all their wealth at 0 to buy the risky asset Y, since by paying p_Y now they get something with expected payoff next period greater than p_Y and they are not impatient. Those who think

$$\gamma_U^h d_U + (1 - \gamma_U^h) d_D < p_Y$$

will sell their share of the asset and buy either consumption good (between which they are indifferent).

Under the assumption that γ_U^h is strictly monotically increasing and continuous in h, there must be a unique agent h^* who is indifferent between W and Y. We call him the marginal agent. Those above h^* will spend all their money on Y, and those below h^* will spend all their money on W. The presence of the marginal agent makes it easy to describe and compute equilibrium.

Without borrowing, equilibrium (h^*, p_Y) must solve two equations

$$\gamma_U^{h^*} d_U + (1 - \gamma_U^{h^*}) d_D = p_Y$$

$$(1 - h^*)(1 + p_Y) = p_Y$$

where the first says that the marginal agent h^* is indifferent between W and Y, and the second equation says that if the top $(1-h^*)$ agents spend all their income they should just be able to afford to buy the one unit outstanding of Y.

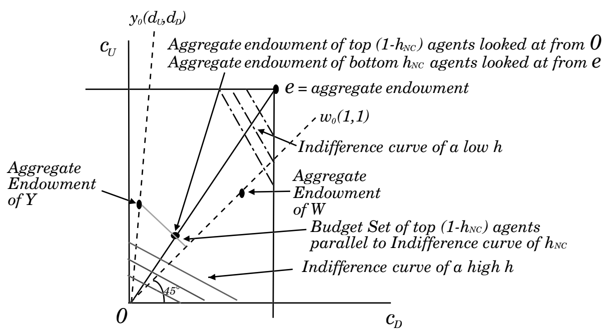
In the numerical examples that follow we shall always suppose that every agent owns one unit of the risky asset at time 0 and also one unit of the warehousable consumption good at time 0, $e_0^h = (e_{01}^h, e_{02}^h, e_{03}^h) = (e_{C_o}^h, e_{Y_o}^h, e_{W_o}^h) = (0, 1, 1)$, and that the output from the risky asset is 1 in the up state U and 0.2 in the down state D. The endowments and asset payoffs are thus

$$e^h = (e_{C_o}^h, e_{Y_o}^h, e_{W_o}^h, e_{C_U}^h, e_{C_D}^h) = (0, 1, 1, 0, 0)$$

$$(d_U, d_D) = (0, 0.2)$$

Suppose $\gamma_U^h = h$ for all h. Then solving the system of two equations gives equilibrium $(h^*, p_Y) = (0.596, 0.677) \approx (0.60, 0.68)$. Agent $h = 0.60$ values the asset at $0.68 = 0.60(1) + 0.40\ (0.2)$. Each agent above 0.60 will spend all his 1.68 of wealth on asset Y. The total cost of Y is 0.68, and indeed $0.40(1.68) = 0.67 \approx 0.68$ units in aggregate. Since the market for risky assets clears at time 0, and everybody is optimizing, by the Walras Law, the market for all the other goods must clear as well and this is the equilibrium with no borrowing. In this equilibrium agents are indifferent to storing or consuming right

Figure 7. No Credit Equilibrium



Source: Author's elaboration.

away, so we can describe equilibrium as if everyone warehoused and postponed consumption by taking

$$p = 0.68$$

$$(c_0^h, y_0^h, w_0^h, c_U^h, c_D^h) = (0, 2.5, 0, 2.5, 0.5) \text{ for } h \geq 0.60$$

$$(c_0^h, y_0^h, w_0^h, c_U^h, c_D^h) = (0, 0, 1.68, 1.68, 1.68) \text{ for } h < 0.60.$$

Similarly if agents are more optimistic, and $\gamma_U^h = 1 - (1-h)^2 > h$ for all $h \in (0,1)$, then equilibrium $(h^*, p_Y) = (0.545, 0.835)$. On the other hand, if agents are more pessimistic and $\gamma_U^h = 1 - (1-h)^{0.1} < h$ for all $h \in (0,1)$, then equilibrium $(h^*, p_Y) = (0.764, 0.308)$.

1.3 Arrow Debreu Equilibrium

If agents can commit to delivering fully on state contingent promises, then we get Arrow Debreu equilibrium. Arrow Debreu equilibrium is defined by Arrow prices (π_U, π_D) of the promise to deliver one unit of the consumption good in U, and the promise to deliver one unit in D, together with consumption $(c_0^h, w_0^h, c_U^h, c_D^h)_{h\in H}$ such that supply equals demand

$$\sum_{h\in H}(c_0^h + w_0^h) = \sum_{h\in H}(e_{C_o}^h + e_{W_o}^h)$$

$$\sum_{h\in H} c_U^h = d_U \sum_{h\in H} e_{Y_o}^h + 1\sum_{h\in H} w_0^h + \sum_{h\in H} e_{C_U}^h$$

$$\sum_{h\in H} c_D^h = d_D \sum_{h\in H} e_{Y_o}^h + 1\sum_{h\in H} w_0^h + \sum_{h\in H} e_{C_D}^h$$

and such that each agent h is choosing $(c_0^h, w_0^h, c_U^h, c_D^h)$ to maximize $U^h(c_0, c_U, c_D)$ such that

$$c_0 + \pi_U c_U + \pi_D c_D \leq (e_{C_o}^h + e_{W_o}^h) + \pi_U(e_U^h + d_U e_{Y_o}^h) + \pi_D(e_D^h + d_D e_{Y_o}^h)$$

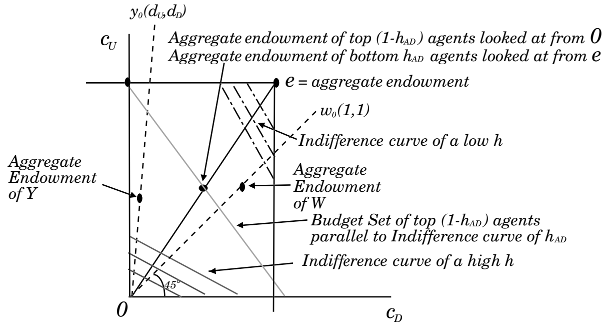
For the economy with a continuum of risk neutral agents who do not discount the future, it is evident that again there must be a marginal buyer h^* such that the agents $h > h^*$ spend all their wealth on c_U and the agents $h < h^*$ spend all their wealth on c_D All the time 0 goods will be warehoused to the future.

Taking endowments $e^h = (0, 1, 1, 0, 0)$ and risky asset payoffs $(d_U, d_D) = (1, 0.2)$ as before, total consumption in U must be 2 and in D it must be 1.2. Suppose $\gamma_U^h = h$ for all h. Then the Arrow Debreu equilibrium is $(h^*, \pi_U, p_Y) = (0.436, 0.436, 0.549) \approx (0.44, 0.44, 0.55)$. Agent $h = 0.44$ values the asset at $0.55 = 0.44(1) + 0.56(0.2)$. Every agent above 0.44 will buy as much as he can afford of the Up Arrow security. Each of these agents can spend 1.55, hence spending 0.56 $(1.55) = 0.87$ in aggregate. Since the cost of all the Arrow up is 2 $(0.436) = 0.87$, the markets clear.

Similarly if agents are more optimistic, and $\gamma_U^h = 1 - (1-h)^2$ for all h then equilibrium $(h^*, \pi_U, p_Y) = (0.33, 0.55, 0.64)$ On the other hand, if agents are more pessimistic and $\gamma_U^h = 1 - (1-h)^{0.1}$ for all h then equilibrium $(h^*, \pi_U, p_Y) = (0.783, 0.142, 0.314)$.

Observe that the asset price in the no borrowing equilibrium can be higher than the Arrow Debreu asset price. Thus when $\gamma_U^h = h$, the Arrow Debreu price is higher $0.68 > 0.55$ and when $\gamma_U^h = 1 - (1-h)^2$ the Arrow Debreu price is also higher, $0.83 > 0.64$. But when $\gamma_U^h = 1 - (1-h)^{0.1}$, the Arrow Debreu price is lower $0.308 < 0.314$. The difference between the two economies is essentially that in the no borrowing economy, there is also no short selling; with short selling of both assets (and delivery fully guaranteed) we would get the Arrow Debreu outcome. If short selling were allowed, the agents

Figure 8. Arrow Debreu Equilibrium



Source: Author's elaboration.

who thought one of the assets was overvalued would sell it short. That can sometimes lower the price of Y, but it can other times lower the price of W.

1.4 Collateral Equilibrium

When credit markets are created, the first question that arises is why should borrowers keep their promises? In the Arrow Debreu model, the implicit assumption is made that anyone who defaults faces an infinite penalty. We shall now suppose to the contrary that no penalties are available, but that there is a state-run court system that is able to seize pledged collateral in case of default and turn it over to he lender.

1.4.1 Collateral

We shall restrict attention to loans that are non-contingent, that is that involve promises of the same amount j in both states. We have not yet determined how much people can borrow or lend. In conventional economics they can do as much of either as they like, at the going interest rate. But in real life lenders worry about default. Suppose we imagine that the only way to enforce deliveries is through collateral. A borrower can use one unit of the asset Y itself as collateral, so that if he defaults the collateral can be seized.[2] Of course a lender realizes that if the promise is j in both states, then with no-recourse collateral he will only receive

$\min(j, d_U)$ if good news

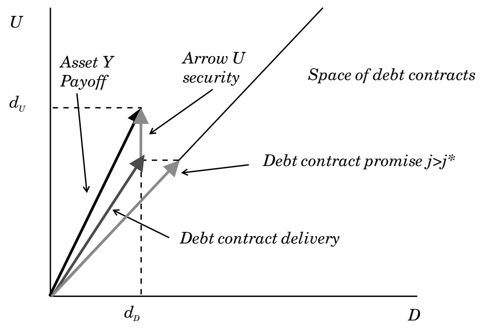
$\min(j, d_D)$ if bad news

Observe that because the owner of the collateral has no influence on the cash flows of the asset, and with no recourse collateral and one period loans, every agent delivers the same on a given contract, namely the promise or the collateral, whichever is worth less. The

2. The other durable good W could also be used as collateral. But since its payoff is the same in both states, and the coontracts are all non-contingent, nobody would ever both to borrow on it. They could simply sell the asset to raise cash. In the case of Y, borrowing on Y gives a net payoff that is different from simply holding Y.

loan market is thus completely anonymous; there is no role for asymmetric information about the agents because every agent delivers the same way. Lenders need only worry about the collateral, not about the identity or actions of the borrowers.

Figure 9. Contract Promises and Deliveries



Source: Author's elaboration.

The introduction of collateralized loan markets introduces two more parameters: how much can be promised j, and at what interest rate r? At first glance there seems to be only one additional market clearing condition, namely, that demand equals supply for loans. How can one equation determine two variables?

1.4.2 The credit surface

Before 1997 there had been virtually no work on equilibrium margins. Collateral was discussed almost exclusively in models without uncertainty (as in Kiyotaki and Moore, 1997), or in corporate finance models in which moral hazard reasons like the potential theft of loans restrained borrowing (as in Holmström and Tirole, 1997). But the 2007–09 crisis revealed that massive shifts in collateral rates or leverage occurrred in assets like mortgage securities, in which the owners of the securities had absolutely no influence on the cash flows, or spccial knowledge of the cash flows. Even now the few writers who try to make collateral endogenous in general equilibrium do so by taking an ad hoc measure of risk, like volatility or value at risk, and assume that the margin is some arbitrary function of the riskiness of the repayment.

It is not surprising that economists have had trouble modeling equilibrium haircuts or leverage. We have been taught that the only equilibrating variables are prices. It seems impossible that the demand equals supply equation for loans could determine two variables.

The key idea, as shown in Geanakoplos (1997), is to think of many loans, not one loan. Irving Fisher and then Ken Arrow taught us to index commodities by their location, or their time period, or by the state of nature, so that the same quality apple in different places or different periods might have different prices. So we must index each promise by its collateral. A promise of $j = d_D$ backed by Y is different from a promise of $j = d_D$ backed by 1/2 of Y. The former delivers d_D in both states, but the latter might deliver d_D in the good state (if $d_U \geq 2d_D$) and $(1/2)d_D$ in the bad state. Doubling the promise does not double the payoff. The collateral matters.

Conceptually we must replace the notion of contracts as promises with the notion of contracts as ordered pairs of promises and collateral, so that each ordered pair-contract will trade in a separate market, with its own price.

$$Contract_j = (Promise_j, Collateral_j) = (A_j, C_j)$$

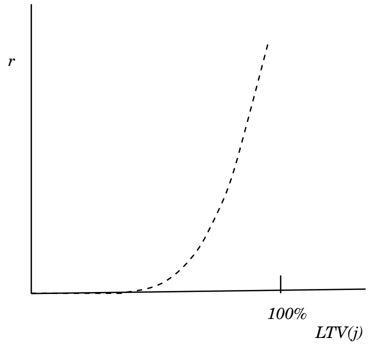
Though the contract payoffs are not homogeneous in the promise with a fixed collateral, the payoffs are indeed homogeneous in the ordered pair. Doubling the promise and the collateral *does* double the payoff of the contract. Trading via the former contract is the same as trading through the latter contract; only the units change. So without loss of generality, we can always normalize the collateral. In our example we shall focus on contracts in which the collateral C_j is simply one unit of Y.

So let us denote by j the promise of j in both states in the future, backed by the collateral of one unit of Y. We take an arbitrarily large set J of such assets, but include $j = d_D = 0.2$. Each contract j type trades at its own price π_j.

Given the price π_j, and given that the promises are all non-contingent, we can always compute the implied nominal interest rate as $1 + r_j = j/\pi_j$. When the collateral is so big that there is no default, $\pi_j = j/(1 + r)$, where r is the riskless rate of interest. But when there is default, the price cannot be derived from the riskless interest rate alone.

In the end we have a menu of contracts, each trading for a different price or, equivalently, a different interest rate. The amalgam of all contracts traces out a surface if we think of the terms of the contract as the argument and the interest rate as a function of these terms. I call this the credit surface. In standard monetary theory we describe credit conditions by the riskless interest rate. The riskless interest rate appears on one end of the credit surface, where the collateral is very big compared to the promise. But credit, and thus activity in the economy, often relies more on the parts of the credit surface that lie beyond the riskless interest rate.

Figure 10. Credit Surface



Source: Author's elaboration.

1.4.3 Collateral budget set and equilibrium

We must distinguish between sales $\varphi_j > 0$ of these collateralized promises (that is borrowing) from purchases $\theta_j > 0$ of these promises (that is lending). The two differ more than in their sign. A sale of a promise obliges the seller to put up the collateral, whereas the buyer of the promise does not bear that burden. The marginal utility of buying a promise will often be much less than the marginal disutillity of selling the same promise, at least if the agent does not otherwise want to hold the collateral.

We can describe the budget set formally with our extra variables.

$$B^h(p_Y, \pi) = \{(c_0, y_0, (\theta_j, \varphi_j)_{j \in J}, w_0, c_U, c_D) \in \mathbb{R}^2_+ \times \mathbb{R}^{2J}_+ \times \mathbb{R}^3_+ : \tag{1}$$

$$c_0 + w_0 + p_Y(y_0 - e^h_{Y_o}) + \sum_{j=1}^{J}(\theta_j - \varphi_j)\pi_j \leq (e^h_{C_o} + e^h_{W_o})$$

$$\sum_{j=1}^{J} \max(\varphi_j, 0) \le y_0 \tag{2}$$

$$c_U - e^h_{C_U} \le 1w_0 + d_U y_0 + \sum_{j=1}^{J} (\theta_j - \varphi_j)\min(d_U, j) \tag{3}$$

$$c_D - e^h_{C_D} \le 1w_0 + d_D y_0 + \sum_{j=1}^{J} (\theta_j - \varphi_j)\min(d_D, j)\}. \tag{4}$$

The first inequality says that expenditure on consumption goods (perishable and warehousable) plus net expenditure on the asset Y plus net expenditure on contracts must be less than or equal to the value of the consumption good endowments. The seond inequality describes the crucial collateral or leverage constraint. Each promise must be backed by collateral, and so the sum of the collateral requirements across all the promises must be met by the Y on hand. The last two equations show the wealth carried into states U and D.

Equilibrium is defined exactly as before, except that now we must have market clearing for all the contracts $j \in J$ Equilibrium is defined by the price of Y and the contract prices (p_Y, π) and agent choices $(c^h_0, y^h_0, (\theta^h_j, \varphi^h_j)_{j \in J}, w^h_0, c^h_U, c^h_D$ in $B^h(p_Y, \pi)$ that maximize U^h defined above such that all markets clear

$$\sum_{h \in H} (c^h_0 + w^h_0) = \sum_{h \in H} (e^h_{C_o} + e^h_{W_o})$$

$$\sum_{h \in H} y^h_0 = \sum_{h \in H} e^h_{Y_o}$$

$$\sum_{h \in H} \theta^h_j = \sum_{h \in H} \varphi^h_j, \quad \forall j \in J$$

$$\sum_{h \in H} c^h_U = d_U \sum_{h \in H} e^h_{Y_o} + 1 \sum_{h \in H} w^h_0 + \sum_{h \in H} e^h_{C_U}$$

$$\sum_{h \in H} c^h_D = d_D \sum_{h \in H} e^h_{Y_o} + 1 \sum_{h \in H} w^h_0 + \sum_{h \in H} e^h_{C_D}$$

1.4.4 Equilibrium leverage

In equilibrium we can define the loan to value (LTV) of each contract by the ratio of the borrowed amount to the value of the collateral

$$LTV(j) = \frac{\pi_j}{p_Y}$$

The loan to value of the collateral Y is the weighted average (according to trading volume) of the leverage on each contract that uses Y as collateral

$$LTV(Y) = \frac{\sum_{h \in H} \pi_j \phi_j^h}{\sum_{h \in H} p_Y \phi_j^h}$$

Equilibrium thus determines the interest rate on each contract, and the LTV of each contract and the asset.

Surpisingly, we shall find that when there are only two states, then all the traded contracts have the same interest rate, and for each asset, every contract using it as collateral has the same loan to value.

Consider again the our numerical example where

$$e^h = (e^h_{C_o}, e^h_{Y_o}, e^h_{W_o}, e^h_{C_U}, e^h_{C_D}) = (0, 1, 1, 0, 0)$$

$$(d_U, d_D) = (0, 0.2)$$

Let $\gamma_U^h = h$ for all $h \in H = [0,1]$. Geanakoplos (2003) proved that there is a unique equilibrium, which we shall describe momentarily. In that equilibrium, the only asset that is traded is $((0.2, 0.2),1)$, namely, $j = 0.2$. All the other contracts are priced, but in equilibrium neither bought nor sold. Furthermore, there is a marginal buyer $h^* = 0.69$ who is indifferent to buying the asset Y and every contract j. Their prices can therefore be computed by using state prices corresponding to the value the marginal buyer $h^* = 0.69$ attributes to them. The price of the asset is therefore

$$P_Y = 0.69(1) + 0.31\,(0.2) = 0.75$$

Similarly the price of the contracts are calculated as

$$
\begin{aligned}
\pi_j &= 0.69 \min(1,j) + 0.31 \min(0.2,j) \\
1 + r_j &= j/\pi_j \\
\pi_{0.2} &= 0.69\ (0.2) + 0.31\ (0.2) = 0.2 \\
1 + r_{0.2} &= 0.2/0.2 = 1.00 \\
\pi_{0.3} &= 0.69\ (0.3) + 0.31\ (0.2) = 0.269 \\
1 + r_{0.3} &= 0.3/0.269 = 1.12 \\
\pi_{0.4} &= 0.69\ (0.4) + 0.31\ (0.2) = 0.337 \\
1 + r_{0.4} &= 0.4/0.337 = 1.19
\end{aligned}
$$

Thus an agent who wants to borrow 0.2 using one house as collateral can do so at 0% interest. An agent who wants to borrow 0.269 with the same collateral can do so by promising 12% interest. An agent who wants to borrow 0.337 can do so by promising 19% interest. The puzzle of one equation determining both a collateral rate and an interest rate is resolved; each collateral rate corresponds to a different interest rate. It is quite sensible that less secure loans with higher defaults will require higher rates of interest.

The surprise is that in this kind of example, with only one dimension of risk and one dimension of disagreement, only one margin will be traded! Everybody will voluntarily trade only the $j = 0.2$ loan, even though they could all borrow or lend different amounts at any other rate.

How can this be? Agent $h = 1$ thinks for every 0.75 he pays on the risky asset, he can get 1 for sure. Wouldn't he love to be able to borrow more, even at a slightly higher interest rate? The answer is no! In order to borrow more, he has to substitute say a 0.4 loan for a 0.2 loan. He would then deliver the same amount in the bad state D, but deliver more in the good state U, in exchange for getting more at the beginning. But that is not rational for him. He is the one convinced the good state U will occur, so he definitely does not want to pay more just where he values money the most.[3]

3. More precisely, buying Y while simultaneously using it as collateral to sell any non-contingent promise of at least 0.2 is tantamount to buying up Arrow securities at a price of 0.69 per unit of net payoff in state U. So $h > 0.69$ is indifferent to trading on any of the loan markets promising at least 0.2. By promising 0.4 per unit of Y instead of 0.2 he simply is buying fewer of the up Arrow securities per contract (because he must deliver more in the up state), but he can buy more contracts (since he is receiving more money at date 0). He can accomplish exactly the same thing selling less 0.2 promises.

The lenders are people with $h < 0.69$ who do not want to buy the asset. They are lending instead of buying the asset because they think there is a substantial chance of bad news. It should be no surprise that they do not want to make risky loans, even if they can get a 19% rate instead of a 0% rate, because the risk of default is too high for them. Indeed the risky loan is perfectly correlated with the asset which they have already shown they do not want. Why should they give up more money at time 0 to get more money in a state U that they think will not occur? If anything, these pessimists would now prefer to take the loan rather than give it. But they cannot take the loan, because that would force them to hold the collateral to back their promises, which they do not want to do.[4]

Thus the only loans that get traded in equilibrium involve margins just tight enough to rule out default. That depends of course on the special assumption of only two outcomes. But often the outcomes lenders have in mind are just two. And typically they do set haircuts in a way that makes defaults very unlikely. Recall that in the 1994 and 1998 leverage crises, not a single lender lost money on repo trades. In the massive crisis of 2007 only a few tens of millions of dollars of repo defaulted, out of trillions loaned. Of course in more general models, one would imagine more than one margin and more than one interest rate emerging in equilibrium. The upshot is that equilibrium leverage for the asset Y must be

$$LTV(Y) = \frac{d_D}{(1 + r_{d_D})\, p_Y} = \frac{0.2}{(1+0)(0.75)} = 29\%$$

To summarize, in the usual theory a supply equals demand equation determines the interest rate on loans. In my theory equilibrium often determines the equilibrium leverage (or margin) as well. It seems surprising that one equation could determine two variables, and to the best of my knowledge I was the first to make the observation (in 1997 and again in 2003) that leverage could be uniquely determined in equilibrium. I showed that the right way to think about the problem of endogenous collateral is to

4. More precisely, agents with $h < b$ will want to trade their wealth for as much consumption as they can get in the down state. But on account of the incompleteness of markets, no combination of buying, selling, borrowing on margin and so on can get them more in the down state than in the up state. So they strictly prefer making the 0.2 loan to lending, or borrowing with collateral, any loan promising more than 0.2 per unit of Y.

consider a different market for each loan depending on the amount of collateral put up, and thus a different interest rate for each level of collateral. A loan with a lot of collateral will clear in equilibrium at a low interest rate, and a loan with little collateral will clear at a high interest rate. A loan market is thus determined by a pair (promise, collateral), and each pair has its own market clearing price. The question of a unique collateral level for a loan reduces to the less paradoxical sounding, but still surprising, assertion that in equilibrium everybody will choose to trade at the same collateral level for each kind of promise. I proved that this must be the case when there are only two successor states to each state in the tree of uncertainty, with risk neutral agents differing in their beliefs, but with a common discount rate. More generally, I conjecture that the number of collateral rates traded endogenously will not be unique, but will be robustly much less than the dimension of the state space, or the dimension of agent types.

The following theorem extends my binomial leverage theorem for risk neutral agents to any agents with any kind of discounting. We have not yet introduced the notation needed to state a formal theorem, but we can informally mention the theorem taken from Fostel-Geanakoplos (2014a) that we shall formally state in the next section.

Binomial No Default Theorem: *Consider the two-period two-state economy described above with concave (not just risk-neutral) utilities. Suppose the risky financial asset pays $d_U > d_D$ in the two states. Then any equilibrium is equivalent to another one (in the sense that all consumptions, commodity prices and contract prices are the same) in which the only traded contract using the risky financial asset as collateral promises $j^* = d_D$ in both states. Thus there is no equilibrium default.*

In binomial economies with financial assets (assets that provide no immediate utility to hold them beyond their dividends), all the trade takes place at the unique cusp of the credit surface where the riskless rate is about to become a risky rate.

1.4.5 Risk reduces leverage

Since there is a unique contract picked out by equilibrium in binomial economies, we can easily define equilibrium leverage and see what determines it. The following is taken from Fostel and Geanakoplos (2014a)

Risk-Leverage Theorem for Binomial Economies: *Consider a two period, two state economy such as the one described above. Suppose the risky financial asset pays $d_U > d_D$ in the two states. Then any equilibrium is equivalent to another one (in the sense that all consumptions, commodity prices and contract prices are the same) in which leverage on every loan backed by the risky asset is*

$$LTV(Y) = \frac{\text{worst case return}}{\text{gross riskless rate}} = \frac{d_D/p_Y}{1+r}$$

This follows immediately from the previous theorem because $\pi_{j^*} = d_D/(1+r)$, hence $LTV(Y) = \pi_{j^*}/p_Y = d_D/(1+r)p_Y$.

Thus we have the very important result that risk reduces leverage, where greater risk is defined by a lower worst case return. It is worth noting that this formula does not link leverage with volatility in general. At best, it links leverage with downside volatility. Of course when risks are symmetric, downside volatility and volatility are the same. But in general they are not.

1.4.6 Tight credit markets

One of the most important concepts in macroeconomics is the idea that at certain times credit is too tight or too loose; these are the moments at which the Fed or the Central Bank is often called upon to act by changing interest rates.

What does it mean for credit markets to be tight? That the interest rate is too high? In collateral equilibrium there is a different meaning. Agents who want to borrow more than they have in collateral equilibrium have to put up more collateral or pay a higher interest rate. Observe that in the equilibrium in our example, every agent $h > h^*$ is borrowing at the riskless interest rate $r = 0\%$, but would dearly like to borrow more at the same rate. They cannot because then they would have to pay a higher interest rate, which they would not like to do, or put up more collateral, which they cannot afford (since any collateral purchase requires a positive downpayment).

The tightness of the credit market for any agent h can be measured by the ratio of the gross interest rate he would be willing to promise to borrow an additional dollar (assuming he was also obligated to deliver the same way he already was deliverying on the money he previously borrowed) divided by the gross interest rate he is paying on the money he is borrowing. In the example, this ratio is higher the higher h is. Agent $h = 1$ thinks that by borrowing 75 cents he can make \$1 for sure at U. Hence, he would be willing to pay a 33% interest rate for an additional penny loan, but cannot borrow any more at 0% than he is already borrowing. In order to borrow a penny more, he would be required to pay a higher interest rate on all the money he borrows.[5]

1.4.7 Computing equilibrium: The marginal buyer

Once we know that only one contract will be traded, and that this contract will not involve default and therefore trade at the riskless interest rate, it becomes very easy to compute equilibrium. As was the case with the no credit economy and the Arrow Debreu economy, when there is a continuum of risk neutral agents, there will be a marginal buyer h^* who is just indifferent to buying the asset, and in the collateral economy, also indifferent to buying every contract. Those $h < h^*$ will sell all the Y they have, and those $h > h^*$ will buy all they can with their cash and with the money they can borrow by trading contract $j = d_D$.

And what interest rate would the the lenders $h < h^*$ get? 0% interest, because they are not lending all they have in cash. (They are lending at most $d_D/h^* = 0.2/0.69 = 0.29 < 1$ per person). Since they are not impatient and they have plenty of cash left, they are indifferent to lending at 0%.Competition among these lenders will drive the interest rate to 0%.

More formally, letting the marginal buyer be denoted by $h = h^*$ we can define the equilibrium equations as

$$p_Y = \gamma_U^{h^*} d_U + (1 - \gamma_U^{h^*}) d_D \tag{5}$$

$$p_Y = (1 - h^*)(1 + p_Y) + d_D$$

Let us return to our numerical example where

$$e^h = (e^h_{C_o}, e^h_{Y_o}, e^h_{W_o}, e^h_{C_U}, e^h_{C_D}) = (0, 1, 1, 0, 0)$$

$$(d_U, d_D) = (0, 0.2)$$

5. The attentive reader will notice that we do not allow tranching or senioriry of loans in this survey. I have treated these subjects elsewhere.

Let $\gamma_U^h = h$ for all $h \in H = [0,1]$. Equation (1) says that the marginal buyer h^* is indifferent to buying the asset. Equation (2) says that the price of Y is equal to the amount of money the agents above h^* spend buying it. As we said, the large supply of the durable consumption good, no impatience, and no default implies that the equilibrium interest rate must be 0.

Solving equations (1) and (2) for p_Y and h^* when beliefs are given by $\gamma_U^h = h$ for all $h \in H$, and plugging these into the agent optimization gives equilibrium

$h^* = 0.69$

$(p_Y, r) = (0.75, 0),$

$(c_0^h, y_0^h, (\theta_{d_D}^h, \varphi_{d_D}^h), w_0^h, c_U^h, c_D^h) = (0, 3.2, (0, 3.2), 0, 2.6, 0)$ for $h \geq 0.69$

$(c_0^h, y_0^h, (\theta_{d_D}^h, \varphi_{d_D}^h), w_0^h, c_U^h, c_D^h) = (0, 0, (1.45, 0), 1.45, 1.75, 1.75)$ for $h < 0.69$.

Compared to the previous equilibrium with no leverage, the price rises from 0.69 to 0.75 because the optimists can borrow to buy more. Notice also that even at the higher price, fewer agents hold all the assets (because they can afford to buy on borrowed money).

Equilibrium can be described picturesquely by observing that the asset price must correspond to the valuation of the marginal buyer. The final holders of the asset are all those whose valuation is higher than the marginal buyer's. Leverage raises the asset price because it enables fewer buyers to hold all the assets (since they can purchase not just by spending the cash they have on hand, but also by borrowing), thus raising the marginal buyer. A higher marginal buyer has a higher valuation for the asset.

We can also compute the equilibrium in the case where agents are more optimistic, and $\gamma_U^h = 1 - (1-h)^2 > h$ for all h. Then equilibrium $(h^*, p_Y) = (0.63, 0.89)$. On the other hand, if agents are more pessimistic and $\gamma_U^h = 1 - (1-h)^{0.1} < h$ for all h, then equilibrium $(h^*, p_Y) = (0.83, 0.44)$. In all three cases, the leverage price is higher than the corresponding no credit price and higher than the corresponding Arrow Debreu price.

Before leaving this example, it is worth noting that the final utility of each agent $h < h^*$ is $1 + p_Y$, while the final utility of each agent $h > h^*$ is $\gamma_U^h / \gamma_U^{h^*}(1 + p_Y)$. To see how to derive the latter expression, observe that by leveraging the risky asset one can effectively purchase the up Arrow security. The prices of all assets are determined by h^*, hence, it can easily be verified that the price of one Arrow up security is $\gamma_U^{h^*}$. But the value to h of that security is γ_U^h. Hence the formula.

1.4.8 Leverage raises asset prices

The lesson here is that the looser the collateral requirement, the higher the prices of assets will be. Had we defined another equilibrium by arbitrarily specifying the collateral limit by prohibiting the selling of contracts unless $j \leq k < d_D$, we would have found an equilibrium price p_Y intermediate between the no borrowing price 0.68 and the fully leverage price 0.75. This has not been properly understood by economists. The conventional view is that the lower the interest rate is, then the higher asset prices will be, because their cash flows will be discounted less. But in the example I just described, where agents are patient, the interest rate will be zero regardless of the collateral restrictions (up to 0.2). The fundamentals do not change, but because of a change in lending standards, asset prices rise. Clearly there is something wrong with conventional asset pricing formulas. The higher the leverage, the higher and thus more optimistic the *marginal buyer* is; it is his probabilities that determine value.

We can state this formally as was done in Fostel-Geanakoplos (2013)

Leverage Pricing Theorem: *Consider the two period, two state economy described above, with a riskless numeraire asset and a risky financial asset paying $d_U > d_D$ in the two states, and a continuum of risk neutral agents with strictly monotonic and continuous beliefs γ_U^h, who each begin with the same endowment of the risky and riskless assets. The collateral equilibrium price of the risky asset will always be higher than the no borrowing equilibrium price of the risky asset.*

Putting together the risk-leverage theorem and the leverage-pricing theorem we see that changes in risk affect asset prices, even if all agents are risk neutral. When risk goes up (say from a mean preserving spread in what everybody thinks the asset payoffs will be), leverage on the risky asset will fall. And when leverage falls, its price falls. Conversely, when risk diminishes, leverage rises and asset prices rise.

Historically, the theory predicts that periods of moderation in asset prices lead to higher leverage which leads to higher asset prices, and conversely.

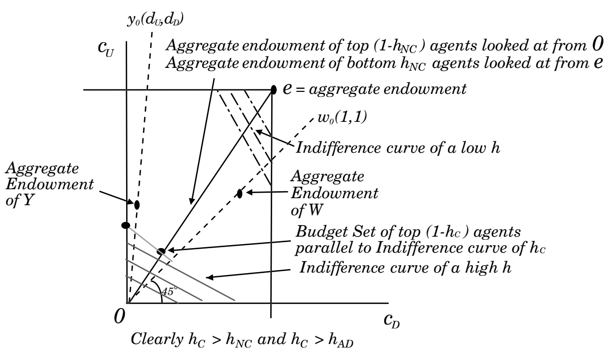
1.4.9 Collateral-Leverage Bubbles

The conventional view of credit markets has been that the need to post collateral in order to borrow to carry out investment (say in education) or to buy essential goods (like housing) lessens demand and therefore reduces the flow compared to a first best Arrow Debreu world in which agents could borrow freely and without limit, as long as they paid back their debts in the end. Our examples show that this intuition is wrong. The following theorem is from Fostel-Geanakoplos (2014b).

Collateral Bubbles Theorem: *Suppose that in the economy described in the Leverage Pricing Theorem there is no endowment of commodities in states U and D. Then the collateral equilibrium price of the risky asset will always be higher than the Arrow Debreu price of the risky asset.*

It follows that if it were possible to produce the risky asset from the riskless asset in period 0, then there would be overproduction instead of underproduction.

Figure 11. Collateral Equilibrium



Source: Author's elaboration.

2. The Collateral Economy in General

Having introduced some of the main ideas of the leverage cycle and collateral equilibrium, we are now in a better position to introduce notation defining a more general collateral economy consisting of many time periods and states of nature, an arbitrary number of perishable goods and durable goods, and one period contracts that can be written on all of them. We use this general model to describe the leverage cycle, which is necessarily part of a dynamic economy.

2.1 Tree of Date-Events

Let S be a finite tree with root 0 and terminal nodes S_T Every node $s \in S\backslash\{0\}$ has a unique immediate predecessor s^*, and every node $s \in S\backslash S_T$ has a set of immediate successors $S(s) = \{t \in S\colon t^* = s\}$. Let $(0, s]$ be the collection of all the points along the path from 0 to s, including s but not 0, and let the time of s, $\tau(s)$, denote the number of points on the path. In a binary tree, every node s $\in S\backslash S_T$ has a set of immediate successors consisting of two elements $S(s) = \{sU, sD\}$.

2.2 Commodities and Assets

At each date-event $s \in S$ the commodity space $\mathbb{R}^{L_S}$ consists of L_S commodities. At the end of trading in the state, each agent h can hold $x_S \in \mathbb{R}_+^{L_S}$ commodities, which provide him utility. These commodities can be perishable or durable or anything in between. To the extent that they are durable, they are sometimes called assets. If they are completely perishable, they will be called goods or perishable commodities. The set of feasible consumption plans is denoted by

$$X = \times_{s \in S} \mathbb{R}_+^{L_s}$$

Given a state $s \in S\backslash S_T$ and an immediate successor $t \in S(s)$, the $L_t \times L_s$ matrix E_t describes the durability of every commodity between s and t. If at node s agent h holds one unit of commodity ℓ after trading is done, then at node t he will have an additional $E_{t\ell'\ell}$ units of each commodity $\ell' \in L_{t.}$ Thus if he holds the bundle $x_s \in \mathbb{R}_+^{L_s}$ at s, he will augment his endowment by $E_t x_s$ at each successor $t \in S(s)$. Note that since every state t has a unique predecessor state, the notation E_t conveys as much information as the more cumbersome notation E_{st} .

Commodity prices are denoted by $p_s \in \mathbb{R}_+^{L_s}$ for all $s \in S$. We denote the set of commodity prices by

$$P = \times_{s \in S} \mathbb{R}_+^{L_s}$$

2.3 Utilities

Each agent h has a utility function

$$U^h : X = \times_{s\in S}\mathbb{R}_+^{L_s} \to \mathbb{R}$$

depending on the holding of all the commodities in every state, that is on consumption plans x We assume U^h is continuous, concave, and weakly monotonic state by state (more of everything in any one state strictly increases utility). Often we specialize to the case of von Neumann Morgenstern utilities u^h. For each $s \in S\backslash\{0\}$ let $\gamma_s^h > 0$ denote the probability that agent h thinks nature will choose s, conditional on having chosen s^*. (Take $\gamma_0^h = 1$). For each $s \in S$ define $\bar{\gamma}_s^h = \Pi_{t\in(0,s]}\gamma_s^h = \dots \gamma_{s^*}^h\gamma_s^h$. Let $0 = \delta_h \leq 1$ denote the discount factor of agent h. We often write

$$U^h(x) = \sum_{s\in S}\bar{\gamma}_s^h \delta_h^{\tau(s)} u^h(x_s)$$

Notice that in our general model we allow for agents to obtain utility from holding every commodity, whether it is perishable or not. Thus in contrast to the simplified two period model described earlier, we allow for nonfinancial asssets such as houses, which give immediate utility and pay dividends in later periods.

2.4 Production

Every agent has access to the same instantaneous, constant returns to scale production technology $Z_s \subset \mathbb{R}^{L_s}$ for each state s. If $z \in Z_s$, then $z_\ell < 0$ means commodity ℓ is an input into production z, and $z_\ell > 0$ means commodity ℓ is an output from production z. We assume that Z_s is a closed, convex, cone and that $0 \in Z_s$. We also assume that there exists some $p \in \mathbb{R}_{++}^{L_s}$ with $p \cdot z \leq 0$ for all $z \in Z_s$. The assumption that Z_s is a cone means that there is constant returns to scale, which allows us to simplify the notation for equilibrium because we can assume that equilibrium production will make zero profits, and so we do not need to keep track of agent income from production. It is well known that the assumption of constant returns to scale can be made without any loss of generality once we have competitive markets and convexity. Define the set of feasible production plans by

$$Z = \times_{s \in S} Z_s$$

2.5 Contracts

At each node $s \in S \setminus S_T$, any agent h can sell a one period contract $j \in J_s$ which promises delivery of $D_{tj} \in \mathbb{R}_+^{L_t}$ in each successor state $t \in S(s)$. The contract must be collateralized by a bundle of commodities $c_j \in \mathbb{R}_+^{L_s}$ at node s. Thus each contract $j \in J = \cup_{s \in S \setminus S_T} J_s$ is characterized by its issuance date $s(j)$, its collateral c_j, and its promise $D_{tj} \in \mathbb{R}_+^{L_t}$ in each successor state $t \in S(s(j))$ of $s(j)$.

There is no punishment for failure to keep promises, except for the confiscation of collateral. Hence actual money delivery per unit promise in each successor state $t \in S(s)$ is given by

$$\bar{D}_{tj} = \min(p_t \cdot D_{tj}, p_t \cdot E_t c_j)$$

Deliveries depend on the future prices p_t; even if the promise D_{tj} and the collateal $E_t c_j$ are non-contingent, the delivery might be if the prices are contingent. The vector of deliveries across contracts in any state s is denoted by $\bar{D}_s \in \Delta_{sj} = \mathbb{R}_+^{J(s^*)}$. The whole vector of deliveries is denoted by

$$\bar{D} \in \Delta = \times_{s \in S} \Delta_s$$

We denote the purchase of contract j by the holding $\theta_j \geq 0$ and the sale (or issuance) of contract j by $\phi_j \geq 0$. We denote the vector of contract purchases in any state $s \in S$ by $\theta_s \in \Theta_s = \mathbb{R}_+^{J(s)}$ and the set of contract purchase plans by

$$\Theta = \times_{s \in S} \Theta_s$$

Similarly we denote the vector of contract sales in any state $s \in S$ by $\varphi_s \in \Phi_s = \mathbb{R}_+^{J(s)}$ and the vector of contract sale plans by

$$\Phi = \times_{s \in S} \Phi_s$$

Contract prices are denoted by π_{sj}. An agent who chooses $\phi_{sj} > 0$ for $j \in J(s)$ is borrowing $\pi_{sj}\phi_{sj}$ dollars in state s and the agent who chooses $\theta_{sj} > 0$ is lending $\pi_{sj}\theta_{sj}$ dollars in state s. We denote the vector of contract prices in state s by $\pi_s \in \Pi_s = \mathbb{R}_+^{J(s)}$ and the set of all contract prices by

$$\Pi = \times_{s \in S} \Pi_s$$

2.6 Budget Set

Assuming $\theta_{0^*} = \phi_{0^*} = 0$, and $x_{0^*}^h = 0$, we define the budget set for each agent h by

$$B^h(p, \pi, \bar{D}) = \{(x, \theta, \phi) \in X \times \Theta \times \Phi : \forall s \in S$$

$$p_s \cdot (x_s - e_s^h - E_s x_{s^*}) + \pi_s \cdot (\theta_s - \phi_s) \leq \bar{D}_s \cdot (\theta_{s^*} - \phi_{s^*})$$

$$\sum_{j \in J} c_j \phi_{sj} \leq x_s\}$$

where

$$\bar{D}_{sj} = \min(p_s \cdot D_{sj}, p_s \cdot E_s c_j)$$

2.7 Collateral Equilibrium

$$(p, \pi, z, \bar{D}, (x^h, \theta^h, \phi^h)_{h \in H}) \in P \times \Pi \times Z \times \Delta \times (X \times \Theta \times \Phi)^H$$

such that

$$\sum_h x_s^h = \sum_h (e_s^h + E_s x_{s^*}^h) + z_s \quad \text{for all } s \in S$$

$$\sum_h \theta_s^h = \sum_h \phi_s^h \quad \text{for all } s \in S$$

$$\bar{D}_{sj} = \min(p_s \cdot D_{sj}, p_s \cdot E_s c_j) \quad \text{for all } s \in S, j \in J_s$$

$$p_s \cdot z_s = 0 \geq p_s \cdot z'_s \quad \text{for all } s \in S, z'_s \in Z_s$$

$$(x^h, \theta^h, \phi^h) \in \arg \max_{(x, \theta, \phi) \in B^h(p, \pi, \bar{D})} U^h(x) \quad \text{for all } h \in H.$$

2.8 Binomial No Default and Leverage Theorem

We now have enough notation in place to formally state a theorem from Fostel and Geanakoplos (2013) about default and leverage for financial assets in binomial economies.

Binomial No Default and Risk-Leverage Theorem: *Consider a collateral equilibrium* $(p, \pi, z, \bar{D}, (x^h, \theta^h, \phi^h)_{h\in H})$ *for a collateral economy described in the last section. Suppose the tree S of date events is binomial. Consider any contract j whose collateral* c_j *does not affect any agent's utility in the issuance date* $s(j)$.[6] *Suppose there is another contract* $j^* \in J$ *with* $s(j^*) = s(j)$ *and some* $\lambda > 0$ *with* $p_t \cdot D_{tj^*} = \lambda p_t \cdot D_{tj} \leq p_t \cdot E_t c_j$ *for all* $t \in S(s)$ *and* $p_t \cdot D_{tj^*} = \lambda p_t \cdot D_{tj} = p_t \cdot E_t c_j$ *for some* $t \in S(s)$. *Then there is another collateral equilibrium* $(p, \pi, z, \bar{D}, (x^h, \bar{\theta}^h, \bar{\phi}^h)_{h\in H})$ *with the same consumptions and prices in which contract j is not traded (unless* $j = j^*$*). In particular, every collateral equilibrium is equivalent to one in which there is no default on contracts collateralized by financial assets. Furthermore, suppose that all contracts j written in state s that use some bundle* c_j *as collateral are non-contingent,* $p_{sU} \cdot D_{sUj} = p_{sD} \cdot D_{sDj}$. *Then the leverage of collateral* c_j *can be taken to be*

$$LTV(c_j) = \frac{1}{1+r_s} \frac{\min(p_{sU} \cdot E_{sU} c_j, p_{sD} \cdot E_{sD} c_j)}{p_s \cdot c_j}$$

where r_s *is the unambiguously defined riskless interest rate in state s. In particular, the loan to value (hence, leverage) on any collateral in state s is inversely related to the worst case return or "risk" of the collateral.*

The theorem shows that in binomial economies we do not need to consider default on loans collateralized by financial assets. The only non-contingent contracts that need to be considered are those that promise the maximum amount that can be delivered for sure in both states. But that does not mean the spectre of default is irrelevant. Indeed, the leverage of any financial asset depends crucially on the possibility of default, so that the more risky the asset's payoffs, the less it can be leveraged.

6. By the definition of collateral economy we have described above, the productivity of the collateral $E_t c_j$ does not depend on who owns it either, for any $t \in S(s)$. Hence we are talking about a financial asset (bundle) c_j.

3. The Leverage Cycle

In the two period economy we already clearly saw how risk can reduce leverage, and how reduced leverage causes asset prices to fall. Conversely, moderations of risk tend to increase leverage and increase asset prices. In the two-period leverage example of section 2 the price of the leveraged risky asset starts off too high in period 0. When bad news occurs and the value plummets in the last period to 0.2, there is a crash. But this is a crash in the fundamentals. There is nothing the government can do to avoid it.

The point of the leverage cycle is that excess leverage followed by excessive deleveraging will cause a crash even before there has been a crash in the fundamentals, and even if there is no subsequent crash in the fundamentals. When the price crashes *everybody* will say it has fallen more than their view of the fundamentals warranted. The asset price is excessively high in the initial period (compared to the first best Arrow Debreu price) because volatility is low and there is too much leverage, and it crashes after just a little bit of bad news, provided the news increases volatility, which leads to deleveraging. The fluctuations in fundamental volatility create fluctuations in leverage which itself creates excess volatility of the asset price. Had leverage been curtailed by government regulation in the initial period, the initial asset price would have been lower and the asset price after the bad news would have been higher, smoothing the cycle.

3.1 A Three-Period Model

Let us consider the same example but with three periods instead of two, taken from Geanakoplos (2003) and Geanakoplos (2010). The state space is now $S = \{0, U, D, UU, DU, DD\}$. Notice that after U there is no uncertainty, because the only successor state is UU, whereas after D there is still uncertainty because there are two successor states DU and DD. If going from 0 to D is bad news, it is also scary bad news because it also means an increase in volatility. Suppose that in the three states $0, U, D$ there are three commodities: the perishable consumption good, risky asset, and durable consumption good C, Y, W as before. The holdings of these three commodities are denoted by $x_s = (x_{s1}, x_{s2}, x_{s3}) = (c_s, y_s, w_s)$, for $s \in \{0, U, D\}$. Suppose there is just one commodity in each state $UU, DU, DD,$ which we think of as the perishable consumption good, and whose holdings we denote by $x_s = c_s, s \in \{UU, DU, DD\}$. Let every agent own one unit of the risky asset at time 0 and also one unit of the warehousable consumption good at time 0, $e_0^h = (e_{01}^h, e_{02}^h, e_{03}^h) = (0, 1, 1)$, and nothing in every other state. But now suppose the asset Y pays off after two periods instead of one period. After good news in either period the asset pays 1 unit of the perishable consumption good at the end, otherwise 0.2 of the perishable consumption good. Thus at UU and DU it pays off 1, and only with two pieces of bad news at DD does the asset pay 0.2.

More precisely

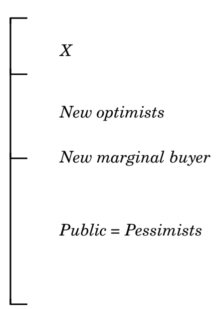
$$E_U = E_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

meaning that one unit of C at time 0 becomes nothing of any of the commodities at U or D (represented by the first column of the matrix), while one unit of Y at time 0 becomes 1 unit of Y at U and D (represented by the second column) and one unit of W becomes 1 unit of W at U and at D (represented by the third column of each matrix). Furthermore,

$$E_{UU} = [\,0 \quad 1 \quad 1\,], E_{DU} = [\,0 \quad 1 \quad 1\,], E_{DD} = [\,0 \quad 0.2 \quad 1\,]$$

meaning that the perishable good at U or D turns into nothing at the terminal nodes (represented by the first column of each matrix), while one unit of Y at U turns into 1 unit of the perishable good at

Figure 12. Leverage Cycle Tree



Source: Author's elaboration.
Leverage Cycle starts before scary news. Uncertanity and disagreement grow from U to D.

UU (represented by the second column of the first matrix), as does one unit of W (represented by the third column of the first matrix), while one unit of Y at D turns into 1 unit of C at DU and only 0.2 units of C at DD (as represented by the second column of the last two matrices), while one unit of W at D turns into 1 unit of C at both DU and DD (as represented by the last column of the final two matrices).

This is a situation in which two things must go wrong (i.e., two down moves) before there is a crash in fundamentals. Investors differ in their probability beliefs over the odds that either bad event happens. The move of nature from 0 to D lowers the expected payoff of the asset Y in every agent's eyes, and also increases every agent's view of the variance of the payoff of asset Y. The news creates more uncertainty, and more disagreement.

As before we suppose that the agents can all turn the durable consumption good into the perishable consumption good at any time, so we describe the intraperiod technology

$$Z_s = \{z = (z_{01}, z_{02}, z_{03}) : z \le (\lambda, 0, -\lambda), \lambda \in \mathbb{R}\} \quad \text{for } s = 0, U, D$$

$$Z_s = \{0\} \quad \text{for } s = UU, DU, DD$$

Suppose again that agents have no impatience, and care only about their expected consumption of the perishable consumption good C. We suppose as before that there is a continuum of agents $h \in [0,1]$ and that

$$U^h(c_0, c_U, c_D, c_{UU}, c_{DU}, c_{DD})$$
$$= c_0 + \gamma_U^h c_U + \gamma_D^h c_D + \gamma_U^h c_{UU} + \gamma_D^h \gamma_{DU}^h c_{DU} + \gamma_D^h \gamma_{DD}^h c_{DD}$$
$$((e_{0c}^h, e_{0y}^h, e_{0w}^h), (e_{Uc}^h, e_{Uy}^h, e_{Uw}^h), (e_{Dc}^h, e_{Dy}^h, e_{Dw}^h), e_{UU}^h, e_{DU}^h, e_{DD}^h)$$
$$= ((0,1,1),(0,0,0),(0,0,0),0,0,0)$$

We suppose that γ_U^h and γ_{DU}^h are strictly increasing in h. Note that agent h assigns only a probability of $\gamma_D^h \gamma_{DD}^h$ to reaching the only state, DD, where the asset pays off 0.2.

3.1.1 Equilibrium

In each state s let the price of the perishable consumption good be normalized to 1. Since the perishable consumption good can be produced one to one from the durable consumption good, the latter must also have a price of 1 in states $s = 0, U, d$. We denote the price of the asset by p_{sy} in each state $s \in S \backslash S_T$.

We suppose that at each state $s \in S \backslash S_T$ it is possible to promise any amount i of the perishable consumption good in both of the following two states sU, sD, using one unit of Y as collateral at s. Denote each such contract by si.

The crucial question again is how much leverage will the market allow at each state s? From the Binomial No Default and Leverage Theorem described in the previous section, it can be shown that in every state s, the only promise that will be actively traded is the one that makes the maximal promise on which there will be no default. Since there will be no default on this contract, it trades at the riskless rate of interest r_s per dollar promised. It will result in equilibrium that the interest rate is zero in every state. Thus at time 0, agents can borrow the minimum of the price of Y at U and at D, for every unit of Y they hold at 0. At U agents can borrow 1 unit of the consumption good, for every unit of Y they hold at U. At D they can borrow only 0.2 units of the consumption good, for every unit of Y they hold at D. In normal times, at 0, there is not very much bad that can happen in the short run. Lenders are therefore willing to lend much more on the same collateral, and leverage can be quite high.

Geanakoplos (2003, 2010) proved that the unique equilibrium in this model is of the following form. At time 0 agents $h \in [a, 1]$ leverage as much as they can to buy all of asset Y. At U their bets pay off and after delivering fully on their loans, they hold their remaining wealth in Y until consumption at UU. At D, however, they owe the totality of the value of their asset holdings. They pay off their debts but are left penniless. At D a new class of buyers $h \in [b, a)$ leverage as much as they can to buy all the assets. The price of the asset tumbles at D not just because the news is bad, but much more importantly, because the marginal buyer drops from a to $b < a$. The drop from a to b is so big because all the agents in $[a, 1]$ are wiped out because they took such huge losses from being so leveraged, and because, at D equilibium *LTV* is so much smaller than at U or than it was at 0, so it requires far more agents to hold the assets, and thus $a - b > 1 - a$.

3.1.2 Finding the equilibrium: The marginal buyers

To see how to find this equilibrium, let b be the marginal buyer in state D and let a be the marginal buyer in state 0. Then we must have

$$p_{DY} = (a-b)(1+p_{0Y}) + 0.2 \tag{6}$$

$$p_{DY} = \gamma^b_{DU} 1 + \gamma^b_{DD}(0.2) \tag{7}$$

$$p_{0Y} = (1-a)(1+p_{0Y}) + p_D \tag{8}$$

$$\frac{\gamma^a_U 1 + \gamma^a_D p_D \frac{\gamma^a_{DU}}{\gamma^b_{DU}}}{p_{0Y}} = \gamma^a_U 1 + \gamma^a_D 1 \frac{\gamma^a_{DU}}{\gamma^b_{DU}} \tag{9}$$

Equation (6) says that all the money spent from the wealth $(1+p_{0Y})$ carried over from 0 by each agent $h \in [b, a)$ plus all the money 0.2 they can borrow using Y as collateral will be spent to buy the single outstanding unit of Y. Equation (7) says that the price at D is equal to the valuation of the marginal buyer b at D. Because he is also indifferent to borrowing, he will then also be indifferent to buying on the margin, as we saw in the collateral section.

Equation (8) is similar to equation (6). It explains the price of Y at 0 must be equal to the expenditure of money used to buy it. Notice that at 0 it is possible to borrow p_D using each unit of Y as collateral. So the top $(1-a)$ agents have $(1-a)(1+p_{0Y}) + p_D$ to spend on the one unit of Y outstanding.

Equation (9) is the most subtle one. It says that the marginal utility at 0 to a of holding one dollar's worth of the durable consumption good, on the right, must be equal to the marginal utility of one dollar of the asset on the left.

To see where the right hand side of equation (9) comes from, observe first that agent a can do better by inventorying the dollar (i.e., warehousing the consumption good by taking $w_0 > 0$) at time 0 rather than consuming it. With probablity γ^a_U, U will be reached and this dollar will be worth one utile. With probability γ^a_D, D will be reached and a will want to leverage the dollar into as big a purchase of Y as possible. As we saw in our two period example, this will result in a gain at D of $\gamma^a_{DU}/\gamma^b_{DU}$. The right hand side is derived similarly.

3.1.3 Crash because of bad news, de-leveraging, and bankrupt optimists

Consider now the case from Geanakoplos (2010) in which $\gamma^h_U = \gamma^h_{UD} = h$ for all $h \in [0,1]$ Plugging that into the equations and solving gives $a = 0.87$, $p_{0Y} = 0.95$, $b = 0.61$, and $p_{DY} = 0.69$. The price of Y at time 0 of 0.95 occurs because the marginal buyer is $h = 0.87$. Assuming the price of Y is 0.69 at D and 1 at U, the most that can be promised at 0 using Y as collateral is 0.69. With an interest rate $r_0 = 0$, that means 0.69 can be borrowed at 0 using Y as collateral. Hence the top 13% of buyers at time 0 can collectively borrow 0.69 (since they will own all the assets), and by adding their own 0.13 of money they can spend 0.82 on buying the 0.87 units that are sold by the bottom 87%. The price is $0.95 \approx 0.82/0.87$.

Why is there a crash from 0 to D? Well first there is bad news. But the bad news is not nearly as bad as the fall in prices. The marginal buyer of the asset at time 0, $h = 0.87$, thinks there is only a $(0.13)^2 = 1.69\%$ chance of ultimate default, and when he gets to D after the first piece of bad news he thinks there is a 13% chance for ultimate default. The news for him is bad, accounting for a drop in price of about $[0.9831(1) + 0.0169\,(0.2)] - [0.87(1) - 0.13(0.2)] \approx 0.986 - 0.896 \approx 9$ points, but it does not explain a fall in price from 0.95 to 0.69 of 26 points. In fact, no agent h thinks the loss in value is nearly as much as 26 points. The biggest optimist $h = 1$ thinks the value is 1 at 0 and still 1 at D. The biggest pessimist $h = 0$ thinks the value is 0.2 at 0 and still 0.2 at D. The biggest loss attributable to the bad news of arriving at D is felt by $h = 0.5$, who thought the value was 0.8 at 0 and thinks it is 0.6 at D. But that drop of 20 points is still less than the drop of 26 points in equilibrium.

The second factor is that the leveraged buyers at time 0 all go bankrupt at D. They spent all their cash plus all they could borrow at time 0, and at time D their collateral is confiscated and used to pay off their debts: they owe 0.69 and their collateral is worth 0.69. Without the most optimistic buyers, the price is naturally lower.

Finally, and most importantly, the margins jump from $(0.95 - 0.69)/0.95 = 27\%$ at 0 to $(0.69 - 0.2)/0.69 = 71\%$ at D. In other words, leverage plummets from $3.6 = 0.95/(0.95 - 0.69)$ to $1.4 = 0.69/(0.69 - 0.2)$.

All three of these factors working together explain the fall in price.

3.1.4 Quantifying the contributions of bad news, deleveraging, and bankruptcy of the optimists

In the crisis of 2007–09 there was bad news, but according to most financial analysts, the price of assets fell much farther than would have been warranted by the news. And indeed as the theory (of 2003!) predicted, there were numerous bankruptcies of the most optimistic mortgage companies, and even of great investment banks.

And the drop in leverage was enormous. The marginal buyer of 2009 was different from the marginal buyer of 2007.

These kind of events had occured before in 1994 and 1998. The cycle was more severe this last time because the leverage was higher, and the bad news was worse.

Of the three symptoms of the leverage cycle collapse, which is playing the biggest role in our example? This is an easy calculation to make, because we can introduce each of the three effects on its own into the model and then see how much the price 0.95 declines.

The bad news has the effect of increasing the probability each agent h assigns to the low payoff of 0.2 at DD from $(1-h)^2$ to $(1-h)$. So we can recalculate equilibrium in the same tree, but with $\gamma^h_{sD} \equiv \sqrt{(1-h)} > (1-h)$ for all $s = 0, U, d$. The result is that at node 0 the price is now 0.79. Thus roughly 60% of the drop in value from 0.95 to 0.69 comes from the bad news itself.

But that still leaves 40% of the drop explainable only by non-fundamentals (or technicals as they are sometimes called). We can decompose this 40% into the part that comes from the bankruptcy and disappearance of the most optimistic buyers, and the rest due to the deleveraging.

In the main example, the most optimistic 13% went bankrupt at D. We can isolate this effect simply by beginning with an economy without these agents. Replacing the set of traders $[0,1]$ with $[0, 0.87]$, and therefore the value 1 with 0.87 in the appropriate equations, one can repeat the calculation and find that the price at the original node is 0.89, a drop of 6 points from the original 0.95, and roughy 20% of the original 26 point drop in the example from 0 to D.

In the main example the deleveraging occurred at D when the maximal promise was reduced to 0.2. We can simulate the deleveraging effect alone by reducing our tree to the old one-period model, but replacing the probability of down of $1-h$ with $(1-h)^2$. In that new model the equilibrium promise at node 0 will be just 0.2, but investors will still assign the 0.2 payoff probability $(1-h)^2$. This gives an initial price for the asset of 0.89. Thus deleveraging also explains about 20% of the price crash.

The roughly linear decomposition of the three factors is due to the linearity of the beliefs $\gamma^h_{sU} = h, \gamma^h_{sD} = 1-h$ in h. In my 2003 paper I analyzed exactly this same model but with more optimistic beliefs because I wanted to avoid this linearity, and also to illustrate a smaller crash consistent with the minor leverage cycle crash of 1998. I assumed $\gamma^h_U = 1-(1-h)^2 = \gamma^h_{DU}$, giving probability $(1-h)^4$ of reaching DD from 0. In that specification there are many investors with γ_{sU}^{h} near to 1, but once h moves far from 1, the decline in optimism happens faster and faster. Solving the four equilibrium equations with this specification of probabilities gives $(p_{0Y}, p_{DY}, a, b) = (0.99, 0.87, 0.94, 0.60)$. The price falls only 12 points from $p_{0Y} = 0.99$ at 0 to $p_{0Y} = 0.87$ at D. Only the top 6% of investors buy at 0, since they can leverage so much, and thus go bankrupt at D. Without them from the beginning, the price would still be 0.99, hence the loss of the top tier itself contributes very little. Bad news alone in that model reduces to the example we just computed at great length, which has a starting price of $p_{0Y} = 0.95$. Deleveraging alone in the 2003 example results in a starting price of $p_{0Y} = 0.98$. Hence the three factors independently add up to much less than the total drop. Thus in the 2003 example it was the feedback between the three causes that explained much of the drop. In the 2010 example, the total drop is very close to the sum of the parts.

3.1.5 Conservative optimists

It is very important, and very characteristic of the leverage cycle, that after the crash, returns are much higher than usual. Survivors of the crash always have great opportunities. One might well wonder why investors in the example do not foresee that there might be a crash, and keep their powder dry in cash (or in assets but without leverage) at 0, waiting to make a killing if the economy goes to D. The answer is that many of them do exactly that.

The marginal buyer at 0 in our first example is $h = 0.87$. He assigns probability $1.69\% = (0.13)^2$ to reach DD. So he values the asset at 0 at more than 0.986, as we saw, yet he is not rushing to buy at the price of 0.95. The reason is that he is precisely looking toward the future. These calculations are embodied in the fourth leverage equilibrium equation. The marginal utility to a of reaching the down state with a dollar of dry powder is not $(1-a)$, but $(1-a)\,(a/b)$ precisely because a anticipates that he will have a spectacular gross expected return of a/b at D.

In fact all the investors between 0.87 and 0.74 are refraining from buying what they regard as an underpriced asset at 0, in order to keep their powder dry for the killing at D. If there were only more of them, of course, there would be no crash at D. But as their numbers rise, so does the price at D, and so their temptation to wait ebbs. It is after all a rare bird who thinks the returns at D are so great, yet thinks D is sufficiently likely to be worth waiting for. This is owing to my assumption that investors who think the first piece of bad news is relatively unlikely (high h), also think the second piece of bad news is relatively unlikely (high h again), even after they see the first piece of bad news. This assumption corresponds to my experience that hedge fund managers generally are the ones saying things are not that bad, even after they start going bad.

3.1.6 Endogenous maturity mismatch

Many authors have lamented the dangers of short term borrowing on long term assets, as we have in this example. It is important to observe that the short term loans I described in the three period model arise endogenously. If long, two period, non-contingent loans were also available, then by the previous arguments, since there are only two outcomes even in the final period, the only potentially traded long term loan would promise 0.2 in every state. But the borrowers would much prefer to borrow 0.69 on the short term loan. So the long term loans would not be traded.

This preference for short term loans is an important feature of real markets. Lenders know that much less can go wrong in a day than in a year, and so they are willing to lend much more for a day on the same collateral than they would for a year. Eager borrowers choose the larger quantity of short term loans, and presto, we have an endogenous maturity mismatch. Endogenous collateral can resolve the puzzle of what causes maturity mismatch.

4. Foreclosure Losses

In this section we introduce the hypothesis that if a good is held as collateral in state s by some agent h, then only he can use the good for production. If a borrower finds himself so far underwater that even after repairs the collateral will not be worth as much as the loan, then he will default without making the repairs, and there will be a social loss because it will then be too late for the lender who confiscates the collateral to make the repairs. This situation becomes much more interesting if some borrowers are efficient enough to make repairs and climb back into the money, and some are not. To include that possibility we must allow for heterogeneous production. Encumbered collateral and heterogeneous production complicate the notation.

The example we present in the next section imagines that if a house is put up as collateral, the owner may be able to improve it by building gardens on its land. Some owners may be better at building gardens than others. Suppose that agent h can build $\alpha(h)$ gardens at a small utility of effort cost. If the debt is j and the house plus $\alpha(h)$ gardens are worth more than j, the owner h will build his gardens and fully repay. But if the house plus $\alpha(h)$ gardens are worth less than j, the owner h will not build any gardens and will default not by $[j -$ (house price + value of $\alpha(h)$ gardens)$]$ but instead by the much bigger amount $[j -$ house price$]$. Whether or not default occurs, we suppose that unencumbering the house takes so much time that the new owner cannot build the gardens. As a result, default will result in a deadweight loss to the economy of missed production.

Lenders of course rationally anticipate that some of their borrowers will become so far underwater that it will be optimal for them to choose not to make repairs that cost less than the increase in value they would bring to the house if they were done. Each lender fully understands that if he lowers j, his borrowers will owe less and so more of them will build gardens and he will get a higher repayment rate. He maintains a high j because he is getting a good return and making fewer loans at a higher rate is less profitable. But he does not take into account that if he and all his brother lenders reduced the size of their loans, the future price of housing would go up, and they would all receive more money back because fewer homeowners would be underwater and more gardens would get built.

4.1 Collateral Encumbrances with Heterogeneous Production

Combining delay with heterogeneity forces us to change the notation from the last section. We assume that every individual has access to his own idiosyncratic technology

$$Z_s^h$$

for each $s \in S$ which is a closed, convex, cone in $\mathbb{R}^{L_s}$ that contains 0. We denote the technology of agent h or the set of all his feasible production plans by

$$Z^h = \times_{s \in S} Z_s^h$$

Let $L_s^C \subset L_s$ denote goods that have been sequestered as collateral. If an agent hasn't put up one of these goods himself as collateral for some contract j he himself wrote, then he cannot use it in production. We require that $Z^h_{s\ell} \geq 0$ for all $\ell \in L^C_s$. This means that if a good $\ell \in L^C_s$ is purchased, freeing it from its encumbrance takes so much time that it is too late to use in production in state s.

We do however allow agents to use their own collateral goods in production. Production from goods that nobody else can use allows for the possibility of profitable production in equilibrium even with constant returns to scale. The damaged house has a low value even if it can be fixed for free, because only its owner can do the fixing. But once he fixes it, he can sell it for a high price. Once we allow for profitable production, we must take care to see which contract gets the profits. At one extreme we could combine all the promises into one total promise, and all the collateral into one big collateral portfolio. But we wish to allow for the possibility that an agent raises money from different lenders, posting separate collateral for each. These collaterals cannot be combined, unless an auxiliary rule is prescribed that spells out which contract has claim on the output. To keep the notation manageable, we suppose the collateral backing contract j cannot be used for any production unless all the output using this collateral is encumbered by contract j.

Suppose an agent holds $E_t c_j \phi_j$ goods as collateral for contract j written in state s. These he can use in production, provided that he does not destroy any value, and that any additional value he creates goes to paying off loan j before he keeps any of it. We formalize this as follows.

We denote the set of possible production plans an agent h has with his goods used as collateral for promise j by

$$Z^{hj} = \times_{t \in S(s(j))} Z^h_t$$

But we limit these plans further by supposing that in each state $t \in S(s)$, $z_t = 0$ or z_t must lie in $\phi_j \mathcal{D}^{hj}_t$ where

$$\mathcal{D}^{hj}_t = \{z \in Z^h_t \colon p_t E_t c^j + p_t \cdot z_t \geq p_t \cdot D_{tj} \ \text{ and } \ z_{t\ell} + [E_t c^j]_\ell \geq 0 \ \forall \ell \in L^C_t\}$$

The first inequality says that if $z_t \neq 0$, then it must add so much value to the collateral that the loan can be fully repaid. The second inequality says that z_t does not use any encumbered goods as inputs except those encumbered by the borrower himself for loan j.

4.1.1 Pooling

Since different agents have different production possibilities, one agent might be able, by virtue of superior productivity, to use his collateral to pay off loan j while leaving a profit for himself, while another agent might choose to produce nothing and so default on loan j. If the lender treats all borrowers as anonymous, he effectively lends to anybody who chooses to borrow via contract j. We represent this formally by considering the whole pool of borrowers.

We let $\bar{D}_{tj}^{h}$ denote the dollars lenders expect to be delivered by agents of type h in state t per unit of contract j sold in state $s = s(j)$. Lenders assume that each dollar they lend will be split among the borrowers in proportion to how much each borrows, that is, if a lender lends 1% of the money lent on contract j (that is if he purchases 1% of contract j sold) then he expects 1% of the deliveries of contract j. We let $\bar{D}_{tj}$ denote the average delivery in state t per unit of contract j sold in state s. An agent who buys contract j in state s is therefore getting $\bar{D}_{tj}$ in each state $t \in S(s)$ per unit of contract j purchased in state s. We shall denote by δ_{tj}^{h} the money deliveries actually made by borrowers of type h on contract j in state t. In equilibrium we shall suppose that lenders are rational and so $\bar{D}_{tj}^{h} = \delta_{tj}^{h}$.

4.2 Foreclosure and Heterogeneous Production Budget Set

We now describe the budget set.

$$B^h(p, \pi, \bar{D}) = \{(x, z, (z^j)_{j\in J}, \theta, \phi, \delta) \in X \times Z^h \times \times_{j\in J} Z^{hj} \times \Phi \times \Theta \times \Delta : \forall s \in S$$

$$p_s \cdot (x_s - e_s^h - E_s x_{s^*}) + \pi_s \cdot (\theta_s - \phi_s) \le p_s \cdot (z_s + \sum_{j\in J(s^*)} z_s^j) + \bar{D}_s \cdot \theta_{s^*} - \delta_s \cdot \phi_{s^*}$$

$$\sum_{j\in J_s} c_j \phi_{sj} \le x_s$$

$z_{s\ell} \ge 0$ if $\ell \in L_s^C$, and for all $j \in J(s^*)$

if $\mathcal{D}_s^{hj} = \emptyset$, then $z_s^j = 0$ and $\delta_{sj} = p_s \cdot E_s c^j$

if $\mathcal{D}_s^{hj} \ne \emptyset$, then $z_s^j \in \phi_s \mathcal{D}_s^{hj}$ and $\delta_{sj} = p_s \cdot D_{sj}\}$

where for all $j \in J(s^*)$

$$\mathcal{D}_t^{hj} = \{z \in Z_t^h : p_t E_t c^j + p_t \cdot z_t \ge p_t \cdot D_{tj} \text{ and } z_{t\ell} + [E_t c^j]_\ell \ge 0 \; \forall \ell \in L_t^C\}$$

4.3 Foreclosure and Heterogeneous Production Equilibrium

$$(p, \pi, \bar{D}, (x^h, z^h, (z^{hj})_{j\in J}, \theta^h, \phi^h, \bar{D}^h)_{h\in H})$$

$$\in P \times \Pi \times \Delta \times (X \times Z^h \times \times_{j\in J} Z^{hj} \times \Theta \times \Phi \times \Delta)^H$$

such that

$$\sum_h x_s^h = \sum_h [(e_s^h + E_s x_{s^*}^h) + (z_s^h + \sum_{j\in J(s^*)} z_s^{hj})] \text{ for all } s \in S$$

$$\sum_h \theta_s^h = \sum_h \phi_s^h \text{ for all } s \in S$$

$$\bar{D}_{sj} = \frac{\sum_h \bar{D}_{sj}^h \varphi_{s^*j}^h}{\sum_h \varphi_{s^*j}^h} \text{ if } \sum_h \varphi_{s^*j}^h > 0 \text{ and } \bar{D}_{sj} \geq p_s \cdot E_s c_j$$

$$(x^h, z^h, (z^{hj})_{j\in J}, \theta^h, \phi^h, \bar{D}^h) \in \arg \max_{(x, z, (z^j)_{j\in J}, \theta, \phi, \delta) \in B^h(p, \pi, \bar{D})} U^h(x) \text{ for all } h \in H.$$

4.4 Example

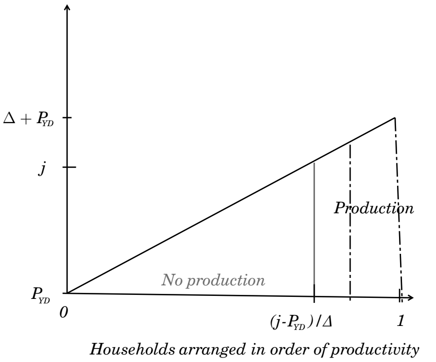
We extend our example from the leverage cycle to include collateral encumbrances and heterogeneous production. So suppose in that model that in the middle period, every agent h can create $\alpha(h)$ units of W with only a very small disutility of effort, where the $\alpha(h)$ are independent, and uniformly distributed on the interval $[0, \Delta]$ where for concreteness we take the parameter $\Delta = 0.1$.

For ease of calculation, we suppose there are just two contracts available, rather than the whole range $j > 0$. In particular, we suppose that the natural contract promise $j^* = p_{DY}$ is still available, as it was in the Leverage Cycle section. Furthermore, we suppose that the contract promise

$$j' = p_{DY} + 0.4\Delta$$

is also available. The most optimistic agents will not be able to resist borrowing more by selling the j' contract than they would be able to

Figure 13. Garden Productivity and Foreclosure Losses



Source: Author's elaboration.

borrow selling the j^* contract. The rational lenders anticipate that 40% of these borrowers will obtain $\alpha(h) < 0.4\Delta$ that are so low that they will default and build no gardens at all rather than pay j'. The price the lenders are willing to pay for the promise j' must reflect this, namely that in the up state U, j' will be fully repaid, while in state D payments will only be

$$0.6(p_{DY} + 0.4\Delta) + 0.4p_{DY} = p_{DY} + \Delta(1 - 0.4)(0.4)$$

The price of the j^* contract will reflect the fact that it is paid back in full in both states; nevertheless the most optimistic agents will prefer to write the j' contract rather than the j^* contract.

We take $\Delta = 0.1$ and solve for equilibrium using the model of the previous section with $\gamma_U^h = h$ for all $h \in H = [0,1]$. In equilibrium there will be four marginal agents h_1, h_2, h_3, h_4. We find that agents $h \in h_1 = [0.959,1]$ buy the risky asset at time 0 for a price $p_{0Y} = 0.993$ by leveraging and borrowing 0.734 using the promise j'. Agents $h \in (h_2 = 0.858, h_1 = 0.959)$ buy the risky asset at time 0 for a price $p_{0Y} = 0.993$ by leveraging and borrowing 0.701 using the promise j^*. Agents $h \in [h_3 = 0.743, h_1 = 0.858)$ buy the risky bonds issued by the most optimistic agents $h \in [h_1, 1]$ and the agents below h_3 hold all the W plus all the safe promises made by the agents $h \in (h_2, h_1)$.

In state U the risky as well as the safe bonds pay off in full. Every Y owner builds a garden, and so $0.05 = 0.5\Delta$ gardens are built.

In state D, the safe bond pays off in full, but there is default on the risky bond. Indeed, the least productive 40% of the agents in the interval $[h_1 = 0.959, 1]$ default on the risky bonds they issued. The other 60% sell off their Y and pay off their risky bonds in full, and with their small surplus of $(0.5)\ (0.6^2)\ \Delta = 0.018$ per unit of Y they borrow more money on the safe bond at D and buy back as much as they can of the risky bonds. Similarly the agents $h \in (h_2 = 0.858, h_1 = 0.959)$ pay off all their safe bond debts, and with their somewhat larger surplus of 0.05 per bond they go on to leverage as much as they can in order to buy back as much Y as they can. Nevertheless, these two groups together will not be able to afford to buy back all the Y.

A more conservative group $h \in [h_4 = 0.626, h_1 = 0.743)$ buys up the remaining Y at D, leveraging as much as they can by selling the riskless promise at D for a price of 0.2. The price $p_{DY} = 0.701$.

Introducing the variable $Q_{0j'}$ to denote the aggregate quantity of risky contracts j' written t time 0, the equilibrium equations are

$$\frac{\gamma_U^{h_1}(1+0.5\Delta-j')+\gamma_D^{h_1}\frac{\gamma_{DU}^{h_1}}{\gamma_{DU}^{h_4}}(0.5)(0.6^2)\Delta}{p_{0Y}-\pi_{j'}}=\frac{\gamma_U^{h_1}(1+0.5\Delta-j^*)+\gamma_D^{h_1}\frac{\gamma_{DU}^{h_1}}{\gamma_{DU}^{h_4}}(0.5)\Delta}{p_{0Y}-\pi_{j^*}}$$

$$\frac{\gamma_U^{h_2}(1+0.5\Delta-j^*)+\gamma_D^{h_2}\frac{\gamma_{DU}^{h_2}}{\gamma_{DU}^{h_4}}(0.5)\Delta}{p_{0Y}-\pi_{j^*}}=\frac{\gamma_U^{h_2}j'+\gamma_D^{h_2}\frac{\gamma_{DU}^{h_2}}{\gamma_{DU}^{h_4}}(j'-(0.4^2)\Delta)}{\pi_{j'}}$$

$$\frac{\gamma_U^{h_3}j'+\gamma_D^{h_3}\frac{\gamma_{DU}^{h_3}}{\gamma_{DU}^{h_4}}(j'-(0.4^2)\Delta)}{\pi_{j'}}=\frac{\gamma_U^{h_3}j^*+\gamma_D^{h_3}\frac{\gamma_{DU}^{h_3}}{\gamma_{DU}^{h_4}}j^*}{\pi_{j^*}}=\frac{\gamma_U^{h_3}+\gamma_D^{h_3}\frac{\gamma_{DU}^{h_3}}{\gamma_{DU}^{h_4}}}{1}$$

$$\frac{\gamma_U^{h_1}+\gamma_D^{h_1}(0.2)}{p_{0Y}}=1$$

$$\frac{\gamma_U^{h_1}+\gamma_D^{h_1}(0.2)}{p_{0Y}}=1$$

$$(1-h_1)(1+p_{0Y})+Q_{0j'}\pi_{j'}=Q_{0j'}p_{0Y}$$

$$(h_1-h_2)(1+p_{0Y})+(1-Q_{0j'})\pi_{j^*}=(1-Q_{0j'})p_{0Y}$$

$$(h_2-h_3)(1+p_{0Y})=Q_{0j'}\pi_{j'}$$

$$Q_{0j'}(0.5)(0.6^2)\Delta+(1-Q_{0j'})(0.5)\Delta+Q_{0j'}(j'-(0.4^2)\Delta)+(h_3-h_4)(1+p_{0Y})+0.2=p_{DY}$$

The first equation says that h_1 is indifferent between buying the risky asset by leveraging with j' or with j^*. Note that he fully takes into account that by borrowing on j' he will deprive himself of producing all the gardens he can at D. The second equation says that h_2 is indifferent to buying Y by leveraging with the riskless bond and buying the risky contract. Notice that he fully takes into account that he will not get fully repaid at D on his j'. The third equation says that h_3 is indifferent between spending on the risky contract j' and the safe contract j^*. The fourth equation says that at D, h_4 is indifferent between Y and W.

The fifth equation says that the top $1 - h_1$ agents buy $Q_{0j'}$ units of the risky asset by issuing $Q_{0j'}$ units of the risky contract j'. The sixth equation says that the next $h_1 - h_2$ agents buy $1 - Q_{0j'}$ units of the risky asset Y by selling $1 - Q_{0j'}$ units of the safe contract j^*. The seventh equation says that the next $h_2 - h_3$ agents buy $Q_{0j'}$ units of the risky contract j' by selling all their W and Y at 0. The *LHS* of the last equation adds all the spending at D on the risky asset Y and asserts it must equal revenue from the sales of Y at D on the *RHS*. The top $1 - h_1$ agents spend all their surplus after paying their debts from the $Q_{0j'}$ risky assets they bought and the next $h_1 - h_2$ agents spend all their surplus after paying their debts from the $1 - Q_{0j'}$ risky assets they bought and the next $h_2 - h_3$ agents spend all their returns from their $Q_{0j'}$ units of the risky contract j' and also the next $h_3 - h_4$ agents spend all the income they carried over from period 0 and in addition they collectively borrow and spend 0.2 by using the risky asset as collateral.

By restraining leverage in period 0, for example by prohibiting trade in j', the leverage cycle can be smoothed out, raising the price at D. Less debt means more income for the upper classes at D, which means a higher price p_{DY}. Also there will be more gardens produced and retained by the upper two classes of buyers, which will increase demand for Y at D, and therefore again lead to a higher price $p_{DY.}$ All agents are better off except the conservative optimists at the top of the $[h_4 = 0.626, h_3 = 0.743)$ range who now do not have as wonderful an opportunity to take advantage of the depressed price of Y at D.

In Geanakoplos and Kubler (2005, 2014) the agents are assumed to be risk averse, and a second source of inefficiency is identified. The risky asset Y becomes riskier the more leverage there is, and its natural buyers still must hold it. Since they are risk averse this puts them in a riskier position. In that model, curtailing leverage at time 0 smoothes the leverage cycle and makes everybody better off.

References

Adrian, T. and H.S. Shin. 2010. "Liquidity and Leverage." *Journal of Financial Intermediation* 19(3): 418−437.

Bernanke, B., M. Gertler and S. Gilchrist. 1996. "The Financial Accelator and the Flight to Quality." *Review of Economics and Statistics* 78(1): 1–15.

————. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, volume 1, edited by J.B. Taylor and M. Woodford, Elsevier.

Brunnermeier, M. and L. Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies* 22(6): 2201–38.

Caballero, R.J. and A. Krishnamurthy. 2001. "International and Domestic Collateral Constraints in a Model of Emerging Market Crises." *Journal of Monetary Economics* 48(3): 513–48.

Fostel, A. and J. Geanakoplos. 2008a. "Collateral Restrictions and Liquidity under-Supply: A Simple Model." *Economic Theory* 35(3): 441–67.

————. 2008b. "Leverage Cycles and the Anxious Economy." *American Economic Review* 98(4): 1211–44.

————. 2013. "Leverage and Default in Binomial Economies: A Complete Characterization" Unpublished manuscript.

Geanakoplos, J. 1997. "Promises, Promises." In *The Economy as an Evolving Complex System* II, edited by. W.B. Arthur, S. Durlauf, and D. Lane. Reading, MA: Addison-Wesley.

————. 2003. "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium." In *Advances in Economics and Econometrics: Theory and Applications*, Eighth World Conference 2: 170–205. Econometric Society Monographs.

————. 2010. "The Leverage Cycle ." In *NBER Macroeconomic Annual 2009* 24: 1–65, edited by D. Acemoglu, K. Rogoff and M. Woodford, Universityof Chicago Press.

Geanakoplos, J. and H. Polemarchakis. 1986. "Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when the Asset Market is Incomplete." In *Uncertainty, Information, and Communication, Essays in Honor of Ken Arrow*, edited by W. Heller, R. Starr, and D. Starrett. Cambridge: Cambridge University Press.

Geanakoplos, J. and F. Kubler. 2005. "Leverage, Incomplete Markets, and Pareto Improving Regulation." Unpublished manuscript.

Geanakoplos, J. and W. Zame. 2009. "Collateralized Security Markets." Unpublished manuscript. Earlier versions 1997, 2002, 2005.

Geanakoplos, J. and W. Zame. 2014. "Collateral Equilibrium, I: A Basic Framework ." *Economic Theory* 56(3): 443 – 492.

Gromb, D. and D. Vayanos. 2002. "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs." *Journal of Financial Economics* 66(2 – 3): 361–407.

Holmström, B. and J. Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *Quarterly Journal of Economics* 112: 663–92.

Kiyotaki, N. and J. Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105(2): 211–48.

Lorenzoni, G. (2008). "Inefficient Credit Booms." *Review of Economic Studies* 75(3): 809–33.

Minsky, H. 1986. *Stabilizing an Unstable Economy*. New Haven: Yale University Press.

Myers, S. 1977. "Determinants of Corporate Borrowing." *Journal of Financial Economics* 5(2): 147–76.

Shleifer, A. and R. Vishny. 1992. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *Journal of Finance* 47(4):134–66.

Simsek, A. 2013. "Belief Disagreements and Collateral Constraints." *Econometrica* 81(1): 1 – 53.

Tobin, J. and S. Golub. 1998. *Money, Credit, and Capital*: Irwin/ McGraw-Hill.